4.3.3 A first look at the runaway greenhouse

We have seen in Chapter 2 that the mass of an atmosphere in equilibrium with a reservoir of condensed substance (e.g. a water ocean) is not fixed. It increases with temperature in accordance with the dictates of the Clausius–Clapeyron relation. If the condensable substance is a greenhouse gas, then the optical thickness τ_∞ increases with temperature. This tends to reduce the *OLR*, offsetting or even reversing the tendency of rising temperature to increase the *OLR*. What are the implications of this for the dependence of *OLR* on surface temperature, and for planetary energy balance? The resulting phenomena are most commonly thought about in connection with the effects of a water ocean on evolution of a planet's climate, but the concept generalizes to any condensable greenhouse gas in equilibrium with a large condensed reservoir. We'll take a first look at this problem here, in the context of the gray gas model.

In the general case, we would like to consider an atmosphere in which the condensable greenhouse gas is mixed with a non-condensable background of fixed mass (which may also have a greenhouse effect of its own). This is the case for water vapor in the Earth's atmosphere, for methane on Titan, and probably also for water vapor in the early atmosphere of Venus. It could also have been the case for mixed nitrogen–CO_2 atmospheres on Early Mars, with CO_2 playing the role of the condensable component. We will eventually take up such atmospheres, but the difficulty in computing the moist adiabat for a two-component atmosphere introduces some distractions which get in the way of grasping the key phenomena. Hence, we'll start with the simpler case in which the atmosphere consists of a pure condensable component in equilibrium with a reservoir (an "ocean," or perhaps a glacier). In this case, the saturated moist adiabat is given by the simple analytic formula Eq. (2.27), obtained by solving the simplified form of the Clausius–Clapeyron relation for temperature in terms of pressure. We have already seen in Chapter 2 that a mixed atmosphere is dominated by the condensable component at high temperatures, so if we are primarily interested in the high-temperature behavior, the use of the one-component condensable atmosphere is not at all a bad approximation.

We write $T(p) = T_0 / \left(1 - \frac{RT_0}{L} \ln \frac{p}{p_0}\right)$, where (p_0, T_0) are a *fixed* reference temperature and pressure on the saturation curve, such as the triple point temperature and pressure. If the surface pressure is p_s, then the surface temperature is $T_s = T(p_s)$. Hence, specifying surface pressure is equivalent to specifying surface temperature in this problem. To keep the algebra simple, we'll assume a constant specific absorption κ, and absorb $\cos\bar{\theta}$ into the definition of κ. Then $\tau_\infty = \kappa p_s/g$, which increases as T_s is made larger. Further, for constant specific absorption, $p/p_0 = (\tau_\infty - \tau')/\tau_0$ where $\tau_0 = \kappa p_0/g$. Now, the choice of the reference temperature and pressure (p_0, T_0) is perfectly arbitrary, and we'll get the same answer no matter what choice we make (within the accuracy of the approximate form of Clausius–Clapeyron we are using). Hence, we are free to set $p_0 = g/\kappa$ so that $\tau_0 = 1$. T_0 then implicitly depends on κ, and becomes larger as κ gets smaller. T_0 is the temperature at the level of the atmosphere where the optical depth measured relative to the top of the atmosphere is unity.

Substituting the one-component $T(p)$ into the integral giving the solution to the Schwarzschild equation, and substituting for pressure in terms of optical thickness, we find

$$\begin{aligned} OLR &= I_+(0)e^{-\tau_\infty} + \int_0^{\tau_\infty} \sigma \frac{T_0^4}{\left(1 - \frac{RT_0}{L}\ln\frac{p}{p_0}\right)^4} e^{-(\tau_\infty - \tau')} d\tau' \\ &= I_+(0)e^{-\tau_\infty} + \sigma T_0^4 \int_0^{\tau_\infty} \frac{1}{\left(1 - \frac{RT_0}{L}\ln\tau_1\right)^4} e^{-\tau_1} d\tau_1 \end{aligned} \tag{4.38}$$

where we have in the second line defined a new dummy variable $\tau_1 = \tau_\infty - \tau'$ as before. The surface temperature enters the expression for *OLR* only through τ_∞, which is proportional to surface pressure. In the optically thin limit, the integral on the right hand side of the expression is small (because τ_∞ is small). This happens at low surface temperatures, because p_s is small when the surface temperature is small. The *OLR* then reduces to the first term, which is approximately $I_+(0)$, i.e. the unmodified upward radiation from the surface. In the optically thick limit, which occurs for high surface temperatures, the term proportional to $I_+(0)$ is negligible, and the second term dominates. This term consists of the flux σT_0^4 multiplied by a non-dimensional integral. Recall that T_0 is a constant dependent on the thermodynamic and infrared optical properties of the gas making up the atmosphere; it does not change with surface temperature. Because of the decaying exponential in the integrand, the integral is dominated by the contribution from the vicinity of $\tau_1 = 0$, and will therefore become independent of τ_∞ for large τ_∞.[1] In the optically thick (high temperature) limit, then, the integral is a function of RT_0/L alone. From this we conclude that the *OLR* becomes independent of surface temperature in the limit of large surface temperature (and hence large τ_∞). This limiting *OLR* is known as the *Kombayashi–Ingersoll limit*. It was originally studied in connection with the long-term history of water on Venus, using a somewhat different argument than we have presented here. We shall use the term to refer to a limiting *OLR* arising from the evaporation of any volatile greenhouse gas reservoir, whether computed using a gray gas model or a more realistic radiation model.

It is readily verified that the integral multiplying σT_0^4 approaches unity as RT_0/L approaches zero. In fact, for typical atmospheric gases L/R is a very large temperature, on the order of several thousand kelvins. Hence, unless the specific absorption is exceedingly small, RT_0/L tends to be small, typically on the order of 0.1 or less. For $RT_0/L = 0.1$, the integral has the value of 0.905. Thus, the limiting *OLR* is essentially σT_0^4. Recalling that T_0 is the temperature of the moist adiabat at one optical depth unit down from the top of the atmosphere, we see that the limiting *OLR* behaves very nearly as if all the longwave radiation were emitted from a layer one optical depth unit from the top of the atmosphere.

Figure 4.3 shows some results from a numerical evaluation of the integral in Eq. (4.38). For small surface temperatures, there is little atmosphere, and the *OLR* increases like σT_s^4. As the surface temperature is made larger, the atmosphere becomes thicker and the *OLR* eventually asymptotes to a limiting value, as predicted. In accordance with the argument given above, the limiting *OLR* should be slightly less than the blackbody flux corresponding to the temperature T_0 found one optical depth down from the top of the atmosphere. This temperature depends on g/κ, which is the pressure one optical depth down from the top. For $g/\kappa = 100\,\mathrm{Pa}$, solving the simplified Clausius–Clapeyron relation for T at 100 Pa yields $T_0 = 250.3\,\mathrm{K}$, whence $\sigma T_0^4 = 222.6\,\mathrm{W/m^2}$; for $g/\kappa = 1000\,\mathrm{Pa}$, $T_0 = 280.1\,\mathrm{K}$ and $\sigma T_0^4 = 349.0\,\mathrm{W/m^2}$. These values are consistent with the numerical results shown in the graph.

Note that for a given atmospheric composition (which determines κ) the Kombayashi–Ingersoll limit depends on the acceleration of gravity. A planet with stronger surface gravity will have a higher Kombayashi–Ingersoll limit than one with weaker gravity. An explicit formula for the dependence on g/κ is obtained by substituting for T_0 using the formula for the single-component saturated adiabat, Eq. (2.27). Thus, the limiting *OLR* can be written in the form

[1] Technically, the integral diverges at extremely large τ_∞, because the denominator of the integrand can vanish. This is an artifact of assuming a constant latent heat and has no physical significance.

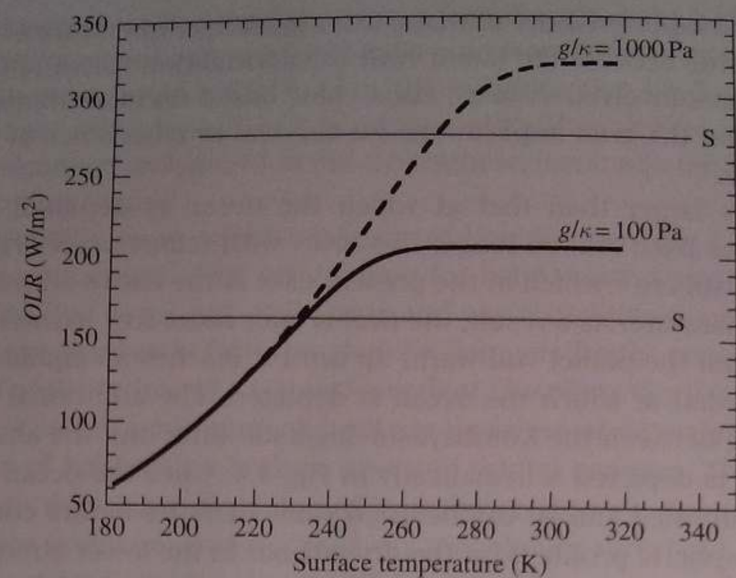

Figure 4.3 *OLR* vs. surface temperature for a one-component gray gas condensable atmosphere in equilibrium with a reservoir. Calculations were done for thermodynamic parameters L and R corresponding to water vapor. Results are shown for two different values of g/κ, where κ is the specific cross-section of the gas and g is the acceleration of gravity. The two horizontal lines marked S indicate two different values of absorbed solar radiation, and their intersections with the *OLR* curves yield the corresponding equilibrium temperatures.

$$OLR_\infty = A(L/R)\sigma T_0^4 = A(L/R)\frac{\sigma (L/R)^4}{\ln(\kappa p^*/g)^4} \tag{4.39}$$

where $A(L/R)$ is the order unity constant discussed previously and $p^* = p_{ref}\exp(L/RT_{ref})$. In the formula for p^*, (p_{ref}, T_{ref}) is any point on the saturation vapor pressure curve, for example the triple point temperature and pressure. p^* is an enormous pressure ($2.3 \cdot 10^{11}$ Pa for water vapor), so Eq. (4.39) predicts that the Kombayashi–Ingersoll limit increases as surface gravity increases, since increasing g makes the logarithm in the denominator smaller. The apparent singularity when $\kappa p^*/g = 1$ is spurious, as the approximations we have made break down long before that value is reached.

We are now prepared to describe the runaway greenhouse phenomenon. Let $(1-\alpha)S$ be the absorbed solar radiation per unit surface area of the planet, and let the limiting *OLR* computed above be OLR_{max}. If $(1-\alpha)S < OLR_{max}$ the planet will come to equilibrium in the usual way, warming up until it loses energy by infrared radiation at the same rate as it receives energy from its star. But what happens if $(1-\alpha)S > OLR_{max}$? In this case, as long as there is still an ocean or other condensed reservoir to feed mass into the atmosphere, the planet cannot get rid of all the solar energy it receives no matter how much it warms up; hence the planet continues to warm until the surface temperature becomes so large that the entire ocean has evaporated into the atmosphere. The temperature at this point depends on the mass and composition of the volatile reservoir. For example, the Earth's oceans contain enough mass to raise the surface pressure to about 100 bars if dumped into the atmosphere in the form of water vapor. The ocean has been exhausted when the saturation vapor pressure reaches this value. Using the simplified exponential form of the Clausius–Clapeyron relation to extrapolate the vapor pressure from the sea-level boiling point (1 bar at 373.15 K), we estimate that this vapor pressure is attained at a surface temperature of about 550 K. This estimate is inaccurate, because the latent heat of vaporization varies appreciably over the range of temperatures involved. A more exact value based on measurements of properties of steam is 584 K, but the grim implications for survival or emergence of life as we know it are largely the same.

At temperatures larger than that at which the ocean is depleted, the mass of the atmosphere becomes fixed and no longer increases with temperature. The greenhouse gas content of the atmosphere – which in the present case is the entire atmosphere – no longer increases with temperature. As a result, the *OLR* is once more free to increase as the surface becomes warmer, and the planet will warm-up until it reaches an equilibrium at a temperature warmer than that at which the ocean is depleted. The additional warming required depends on the gap between the Kombayashi–Ingersoll limit and the absorbed solar radiation. This situation is depicted schematically in Fig. 4.4. Once the ocean is gone, the lower atmosphere is unsaturated and air can be lifted some distance before condensation occurs. The resulting atmospheric profile is on the dry adiabat in the lower atmosphere, transitioning to the moist adiabat at the altitude where condensation starts. The situation is identical to that depicted for CO_2 in Fig. 2.6. Rain will still form in the condensing layer. Much of it will evaporate in the lower non-condensing layer; some of it may reach the ground, but the resulting puddles will tend to evaporate rapidly back into the highly undersaturated lower atmosphere. As surface temperature is made larger, the altitude where condensation sets in moves higher, until at very large temperatures the atmosphere behaves like a non-condensing dry system (albeit one where the entire atmosphere may consist of water vapor).

The runaway greenhouse phenomenon may explain how Venus wound up with such a radically different climate from Earth, despite having started out in a rather similar state. The standard story goes something like this: Venus started with an ocean, and with most of its CO_2 bound up in rocks as is the case for Earth. However, it was just closer enough to the Sun to trigger a runaway greenhouse. Once the entire ocean had evaporated into the atmosphere, there was so much water vapor in the upper atmosphere that it could be

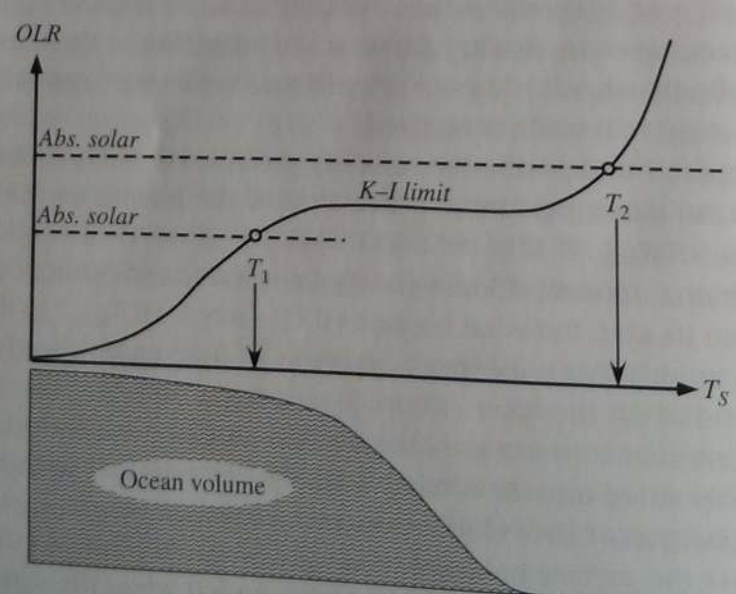

Figure 4.4 Schematic picture of the termination of a runaway greenhouse upon depletion of the volatile reservoir.

broken apart by energetic solar ultraviolet rays, whereafter the light hydrogen could escape to space. The highly reactive oxygen left behind would react to form minerals at the surface. Once there was no more liquid water in play, the reactions that bind up carbon dioxide in rocks could no longer take place (as will be explained in Chapter 8), so all the planet's CO_2 outgassed from volcanism and stayed in the atmosphere, leading to the hot, dry super-dense atmosphere of modern Venus.

Assuming habitability to require a reservoir of liquid water, the Kombayashi–Ingersoll limit for water determines the inner orbital limit for habitability, since if the solar constant exceeds the limiting flux a runaway will ensue and any initial ocean will not persist. It also determines how long it takes before the planet's Sun gets bright enough to trigger a runaway, and thus sets the lifetime of a water-dependent biosphere (Earth's included). Accurate calculations of the Kombayashi–Ingersoll limit are therefore of critical importance to understanding the limits of habitability both in time and orbital position. The gray gas model is not good enough to determine the value of p_0 appropriate to a given gas, and so cannot be used for accurate evaluations of the runaway greenhouse threshold. We can at least say that, all other things being equal, a planet with larger surface gravity will be less susceptible to the runaway greenhouse. This is so because $p_0 = g/\kappa$, whence larger g implies larger p_0, which implies in turn larger $T(p_0)$ and hence a larger limiting *OLR*. This observation may be relevant to the class of extrasolar planets known as "Super Earths."

We will revisit the runaway greenhouse using more realistic radiation physics in Section 4.6. Some consequences of the effects of the stratosphere and of clouds will be brought into the picture in Chapter 5.

The runaway greenhouse phenomenon is usually thought of in conjunction with water vapor, but the concept applies equally well to any situation where there is a volatile reservoir of greenhouse gas, whether it be in solid or liquid form. For example, one could have a runaway greenhouse in association with the sublimation of a large CO_2 ice cap, or in association with the evaporation of a methane or ammonia ocean. In fact, the Kombayashi–Ingersoll limit determines whether a planet would develop a reservoir of condensed substance at its surface (a glacier or ocean), given sustained outgassing of that substance in the absence of any chemical sink. As the gas builds up in the atmosphere, the pressure increases and it would eventually tend to condense at the surface, preventing any further gaseous accumulation. However, the greenhouse effect of the gas warms the surface, which increases the saturation vapor pressure. The Kombayashi–Ingersoll limit tells us which effect wins out as surface pressure increases. Earth is below the threshold for water, so we have a water ocean. Venus is above the threshold for water and CO_2, so both accumulate as gases in the atmosphere (apart from possible escape to space). When we revisit the problem with real gas radiation, we will be able to say whether CO_2 would form a condensed reservoir on Earth or Mars, or CH_4 on Titan, given sustained outgassing in the absence of a chemical sink.

4.3.4 Pure radiative equilibrium for a gray gas atmosphere

For the temperature profiles discussed in Sections 4.3.2 and 4.3.3, the net infrared radiative heating computed from Eq. (4.14) is non-zero at virtually all altitudes; generally the imbalance acts to cool the lower atmosphere and warm the upper atmosphere. In using such solutions to compute *OLR* and back-radiation, we are presuming that convective heat fluxes will balance the cooling and keep the troposphere in a steady state. The upper atmosphere will continue to heat, and ultimately reach equilibrium creating a stratosphere, but in the