Physical basis for quasi-universal relations describing bankfull hydraulic geometry of single-thread gravel bed rivers

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[1] We examine relations for hydraulic geometry of alluvial, single-thread gravel bed rivers with definable bankfull geometries. Four baseline data sets determine relations for bankfull geometry, i.e., bankfull depth, bankfull width, and down-channel slope as functions of bankfull discharge and bed surface median sediment size. These relations show a considerable degree of universality. This universality applies not only within the four sets used to determine the forms but also to three independent data sets as well. We study the physical basis for this universality in terms of four relations, the coefficients and exponents of which can be back calculated from the data: (1) a Manning-Strickler-type relation for channel resistance, (2) a channel-forming relation expressed in terms of the ratio of bankfull Shields number to critical Shields number, (3) a relation for critical Shields number as a function of dimensionless discharge, and (4) a "gravel yield" relation specifying the (estimated) gravel transport rate at bankfull flow as a function of bankfull discharge and gravel size. We use these underlying relations to explore why the dimensionless bankfull relations are only quasi-universal and to quantify the degree to which deviation from universality can be expected. The analysis presented here represents an alternative to extremal formulations to predict hydraulic geometry.

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1. Introduction

[2] Single-thread, alluvial gravel bed rivers represent an important class of natural rivers. Here "gravel bed" is used in a loose sense, and refers to stream reaches with median grain size D_{s50} greater than 25 mm. Many (but by no means all) such river reaches have a distinct channel and floodplain, such that flow spills from the channel onto the floodplain at a well-defined "bankfull" discharge Q_{bf}. For such reaches it is possible to define a "bankfull channel geometry" [*Leopold and Maddock*, 1953; *Leopold et al.*, 1964] in terms of a bankfull width B_{bf}, bankfull depth H_{bf} and down-channel bed slope S.

[3] The variation of these parameters can be cast in terms of power relations of the form

$$B_{bf} = \chi_B Q_{bf}^{n_B} \quad (1a)$$

$$H_{bf} = \chi_H Q_{bf}^{n_H} \quad (1b)$$

$$S = \chi_S Q_{bf}^{-n_S} \quad (1c)$$

(Here n_B and n_H are exponents that are conventionally denoted as b and f, respectively. The notation adopted here is intended to clarify the subsequent analysis.) On the basis of data from gravel bed rivers in Canada, *Bray* [1982] determined the following estimates for the exponents:

$$n_B = 0.527 \quad (2a)$$

$$n_H = 0.333 \quad (2b)$$

$$n_S = 0.342 \quad (2c)$$

This work has been extended by *Hey and Thorne* [1986], who suggest the values 0.52 and 0.39 for n_B and n_H respectively, on the basis of an analysis of British gravel bed streams. Further developments are summarized in Table 2 of *Millar* [2005]. Relations of the form of (1a), (1b), and (1c) are not, however, dimensionally homogeneous, and thus may not reveal the physics underlying the relations. *Parker* [1979], *Andrews* [1984], *Parker and Toro-Escobar* [2002], *Parker et*

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al. [2003] and *Millar* [2005] developed dimensionless forms for bankfull geometry of single-thread gravel bed streams, and *Ashmore and Parker* [1983] developed similar dimensionless relations for anabranches of braided gravel bed streams.

[4] The present analysis has two goals. The first and lesser one is the establishment of quasi-universal dimensionless relations for hydraulic geometry for single-thread, alluvial gravel bed streams. The second and greater one is the specification of a framework that allows them to be derived from a set of underlying physical relations, in a way that does not rely on extremal hypotheses.

2. Governing Parameters

[5] The following parameters are defined for reaches of alluvial, single-thread gravel bed rivers: bankfull discharge Q_{bf}, bankfull width B_{bf}, bankfull depth H_{bf}, down-channel bed slope S, median size D_{s50} of the sediment on the surface of the bed and the acceleration of gravity g. The following relations for hydraulic geometry at bankfull flow are postulated:

$$B_{bf} = f_B\left(Q_{bf}, D_{s50}, g, \textit{other parameters}\right) \tag{3a}$$

$$H_{bf} = f_H\left(Q_{bf}, D_{s50}, g, \textit{other parameters}\right) \tag{3b}$$

$$S = f_S\left(Q_{bf}, D_{s50}, g, \textit{other parameters}\right) \tag{3c}$$

Examples of "other parameters" include gravel supply, the type and density of bank vegetation, bank material type [e.g., *Hey and Thorne*, 1986; *Millar*, 2005] and channel planform. Here the "other parameters" are dropped with the purpose of determining how closely universality can be appproximated with the shortest possible list of governing parameters. Additional parameters are reconsidered later as factors that can contribute to deviation from universality.

[6] Each of (3a), (3b), and (3c) defines a relation involving four parameters (e.g., B_{bf}, Q_{bf}, D_{s50} and g in the case of (3a)) and two dimensions, length and time. The principles of dimensional analysis allow each relation to be expressed in terms of two dimensionless parameters. *Parker* [1979], *Andrews* [1984], *Parker and Toro-Escobar* [2002], *Parker et al.* [2003] have used the following forms:

$$\hat{B} = \hat{f}_{\hat{B}}\left(\hat{Q}\right) \tag{4a}$$

$$\hat{H} = \hat{f}_{\hat{H}}\left(\hat{Q}\right) \tag{4b}$$

$$S = \hat{f}_S\left(\hat{Q}\right) \tag{4c}$$

where

$$\hat{B} = \frac{B_{bf}}{D_{s50}} \tag{5a}$$

$$\hat{H} = \frac{H_{bf}}{D_{s50}} \tag{5b}$$

$$\hat{Q} = \frac{Q_{bf}}{\sqrt{gD_{s50}}\, D_{s50}^2} \tag{5c}$$

Millar [2005] also used similar forms, but included dimensionless measures of the sediment transport rate and bank strength in the formulation.

[7] Here we adopt an alternative but equivalent nondimensionalization for bankfull width and depth, originally suggested by *Bray* [1982]. Defining the dimensionless parameters $\tilde{B}$ and $\tilde{H}$ as

$$\tilde{B} = \frac{g^{1/5} B_{bf}}{Q_{bf}^{2/5}} \tag{6a}$$

$$\tilde{H} = \frac{g^{1/5} H_{bf}}{Q_{bf}^{2/5}} \tag{6b}$$

we seek relations of the following form:

$$\tilde{B} = \hat{f}_{\tilde{B}}\left(\hat{Q}\right) \tag{7a}$$

$$\tilde{H} = \tilde{f}_{\tilde{H}}\left(\hat{Q}\right) \tag{7b}$$

$$S = \hat{f}_S\left(\hat{Q}\right) \tag{7c}$$

More specifically, we anticipate power relations of the form

$$\tilde{B} = \alpha_B \hat{Q}^{n_B} \tag{8a}$$

$$\tilde{H} = \alpha_H \hat{Q}^{n_H} \tag{8b}$$

$$S = \alpha_S \hat{Q}^{n_S} \tag{8c}$$

Note that, as opposed to the coefficients in the relations (1a), (1b), and (1c), which have dimensions that are entirely dependent upon the choice of the exponents, the coefficients in (8a), (8b), and (8c) are dimensionless.

[8] Dimensionless relations involving the forms $\tilde{B}$ and $\tilde{H}$ are equivalent to corresponding relations involving $\hat{B}$ and $\hat{H}$ because according to (5) and (6),

$$\tilde{B} = \hat{B}\hat{Q}^{-2/5} \tag{9a}$$

$$\tilde{H} = \hat{H}\hat{Q}^{-2/5} \tag{9b}$$

Hey and Heritage [1988] have suggested that dimensionless formulations of hydraulic geometry may be subject to

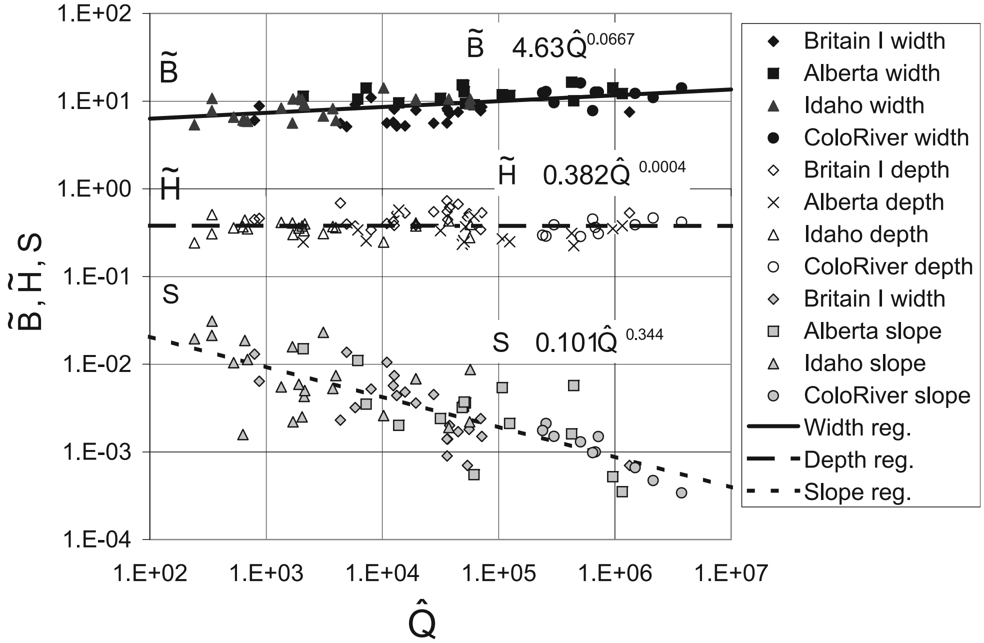

Figure 1. Dimensionless bankfull width $\tilde{B}$, dimensionless bankfull depth $\tilde{H}$, and down-channel bed slope S as functions of dimensionless bankfull discharge $\hat{Q}$. The Alberta, Britain I, Idaho, and Colorado subsets of the baseline data set are discriminated by different symbols. Also shown are power relations derived from regression on the lumped data set.

spurious correlation. The formulation adopted here does not preclude spurious correlation, in that the bankfull discharge Q_{bf} appears in the dependent variables $\tilde{B}$ and $\tilde{H}$ as well as the independent variable $\hat{Q}$. As illustrated in Appendix A, however, dimensionless formulations often require that a dimensioned parameter appear in both the dependent dimensionless grouping and at least one of the independent dimensionless groupings. Failing to adhere to this constraint can lead to physically unsound results.

3. Baseline Data Set

[9] The baseline data set for bankfull geometry of gravel bed streams used here is composed of four subsets. These include (1) 16 stream reaches in Alberta, Canada contained in work by *Kellerhals et al.* [1972] (and identified in more detail by *Parker* [1979]), (2) 23 stream reaches in Britain contained in work by *Charlton et al.* [1978], 23 stream reaches in Idaho, USA [*Parker et al.*, 2003], and (3) 10 reaches of the Colorado River, western Colorado and eastern Utah, USA [*Pitlick and Cress*, 2000], for a total of 72 reaches. These four sets are respectively referred to as "Alberta," "Britain I," "Idaho" and "ColoRiver." The terminology "Britain I" is used because a second set of data from Britain is introduced later.

[10] The baseline data set is available at http://cee.uiuc.edu/people/parkerg/misc.htm. The data for B_{bf}, H_{bf}, S and D_{s50} for each of the 10 reaches of the Colorado River represent medians of values for a larger number of subreaches, as extracted from the compendium in Table A-5 of the appendices of *Pitlick and Cress* [2000]. The data thus differ modestly from the data given in Table 1 of *Pitlick and Cress* [2002], which are based on averages rather than medians.

[11] The parameters of the baseline set vary over the following ranges: bankfull discharge Q_{bf} varies from 2.7 to 5440 m^3/s; bankfull width B_{bf} varies from 5.24 to 280 m; bankfull depth H_{bf} varies from 0.25 to 6.95 m; down-channel bed slope S varies from 0.00034 to 0.031; and surface median grain size D_{s50} varies from 27 to 167.5 mm. Only the data set of *Charlton et al.* [1978] includes measured values for sediment specific gravity. The average value for their 23 reaches is 2.63. In all other cases the sediment specific gravity has been assumed to be the standard value for quartz, i.e., 2.65.

4. Quasi-Universal Relations for Hydraulic Geometry

[12] Figure 1 shows on a single plot $\tilde{B}$, $\tilde{H}$ and S as functions of $\hat{Q}$. The relations define clear trends across four and one half decades of variation of $\hat{Q}$. Standard linear regression yields the following power law forms for dimensionless bankfull hydraulic geometry:

$$\tilde{B} = 4.63 \quad \hat{Q}^{0.0667} \quad i.e.\ \alpha_B = 4.63, \quad n_B = 0.0667 \pm 0.027 \tag{10a}$$

$$\tilde{H} = 0.382 \quad \hat{Q}^{-0.0004} \quad i.e.\ \alpha_H = 0.382, \quad n_H = -0.0004 \pm 0.027 \tag{10b}$$

$$S = 0.101 \quad \hat{Q}^{-0.344} \quad i.e.\ \alpha_S = 0.101, \quad n_S = 0.344 \pm 0.066 \tag{10c}$$

In the above relations, the uncertainties in the exponents were computed at the 95% confidence level. At the 95%

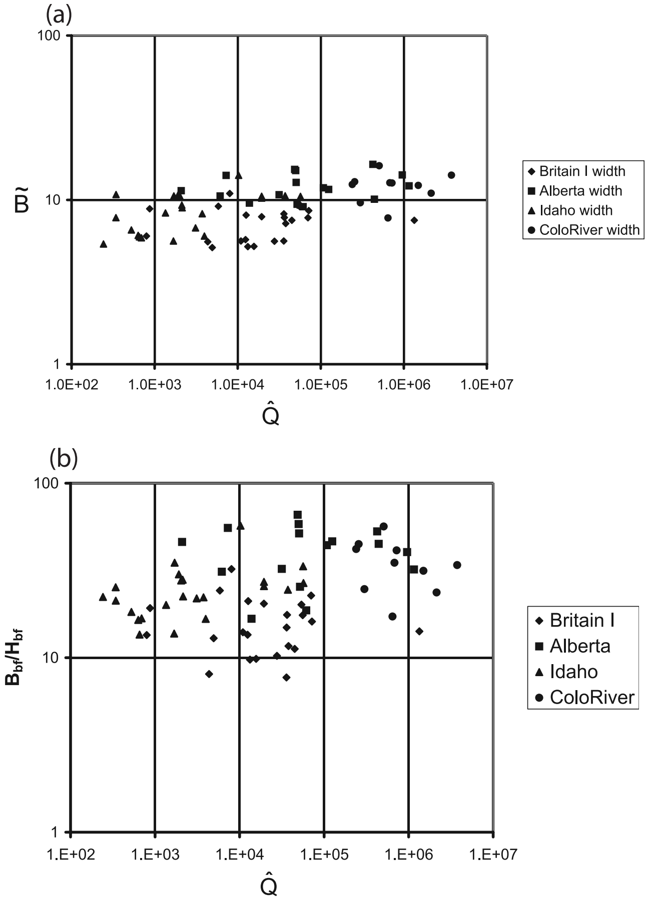

Figure 2. (a) Plot of $\tilde{B}$ versus $\hat{Q}$ for the baseline data set, in which the Alberta, Britain I, Idaho, and Colorado data subsets are distinguished by different symbols. (b) Plot of B_{bf}/H_{bf} versus $\hat{Q}$ for the baseline data set, in which the Alberta, Britain I, Idaho, and Colorado data subsets are distinguished by different symbols.

confidence level the prediction interval is a factor of 3.0 for (10a), 3.0 for (10b) and 14.8 for (10c). These relations turn out upon reduction with (9a, b) to be very close to the relations for $\hat{B}$, $\hat{H}$ and S versus $\hat{Q}$ given by *Parker and Toro-Escobar* [2002] and *Parker et al.* [2003].

[13] The reader should note that ordinary least squares regression has been used here and elsewhere in this paper in preference to, e.g., reduced major axis regression. While there are valid arguments favoring the latter, these arguments do not appear to apply to the present case, as outlined in Appendix B.

[14] Figure 1 and regression relation (10b) indicate that for all practical purposes (10b) can be replaced with constant value

$$\tilde{H} \equiv \tilde{H}_o = 0.382 \tag{11}$$

over the entire range of $\hat{Q}$. Specifically, this yields the dimensional form

$$H_{bf} = \frac{0.382}{g^{1/5}} Q_{bf}^{2/5} \tag{12}$$

The corresponding dimensioned forms for B_{bf} and S are

$$B_{bf} = \frac{4.63}{g^{1/5}} Q_{bf}^{0.4} \left(\frac{Q_{bf}}{\sqrt{gD_{s50}}\, D_{s50}^2} \right)^{0.0667} \tag{13}$$

$$S = 0.101 \left(\frac{Q_{bf}}{\sqrt{gD_{s50}}\, D_{s50}^2} \right)^{-0.344} \tag{14}$$

[15] The exponents of Q_{bf} in (12), (13) and (14) are similar to those found by other authors [e.g., *Millar*, 2005, Table 2], and in particular to those found by authors whose data sets have been included in the present baseline set. Of more significance here is the result that a single set of exponents and coefficients provides a reasonable description of the entire baseline data set, as shown in Figure 1. The data points of the four sets all intermingle one among the other, indicating a substantial degree of universal behavior among data from four distinct geographical regions.

[16] The relations (10a), (11) and (10c) are nevertheless described as "quasi-universal" here because the effects of the "other parameters" in (3) are discernible. Figure 2a illustrates deviation from universality; the Britain I rivers are systematically somewhat narrower than the Alberta rivers. The Britain I rivers are also systematically somewhat deeper than the Alberta rivers, as quantified in terms of the plot of width-depth ratio B_{bf}/H_{bf} versus $\hat{Q}$ of Figure 2b. The role of the width-depth ratio has been emphasized by *Cao and Knight* [1996, 1998] and *Mengoni et al.* [2004].

[17] One reason why the Britain I streams may have lower values of B_{bf}/H_{bf} than the Alberta streams may be the more humid climate and consequent denser bank vegetation in the British streams, so increasing the effective "bank strength" relative to the Alberta streams [e.g., *Charlton et al.*, 1978; *Hey and Thorne*, 1986; *Millar*, 2005]. Another reason may be the likelihood that the British streams have a lower supply of gravel (after normalizing for water supply) than the Alberta streams. Both of these factors are discussed in more detail below.

[18] The scatter in Figures 1 and 2a is at least partly due to different protocols for data collection, as outlined in the original references. It also likely embodies an element of measurement error in the parameters in question. Perhaps the parameter that is most subject to measurement error is the surface median grain size D_{s50}; in most cases the samples of bed material from which it was determined likely did not satisfy the rigorous guidelines of *Church et al.* [1987]. The down-channel bed slope S is subject to error if the reach used to determine it is not sufficiently long. In addition, bankfull width and depth B_{bf} and H_{bf} are subject to error if they are not based on appropriately defined reach averaged characteristics, and bankfull discharge Q_{bf} may be difficult to discern from a rating curve if there is not a clear break in the stage-discharge relationship as the flow spills overbank. A number of these issues are discussed in the careful data compilation of *Church and Rood* [1983].

[19] In Figure 1 data for slope show the most scatter, even though there seem to be no systematic differences among the four data sets. As noted above, part of this scatter may

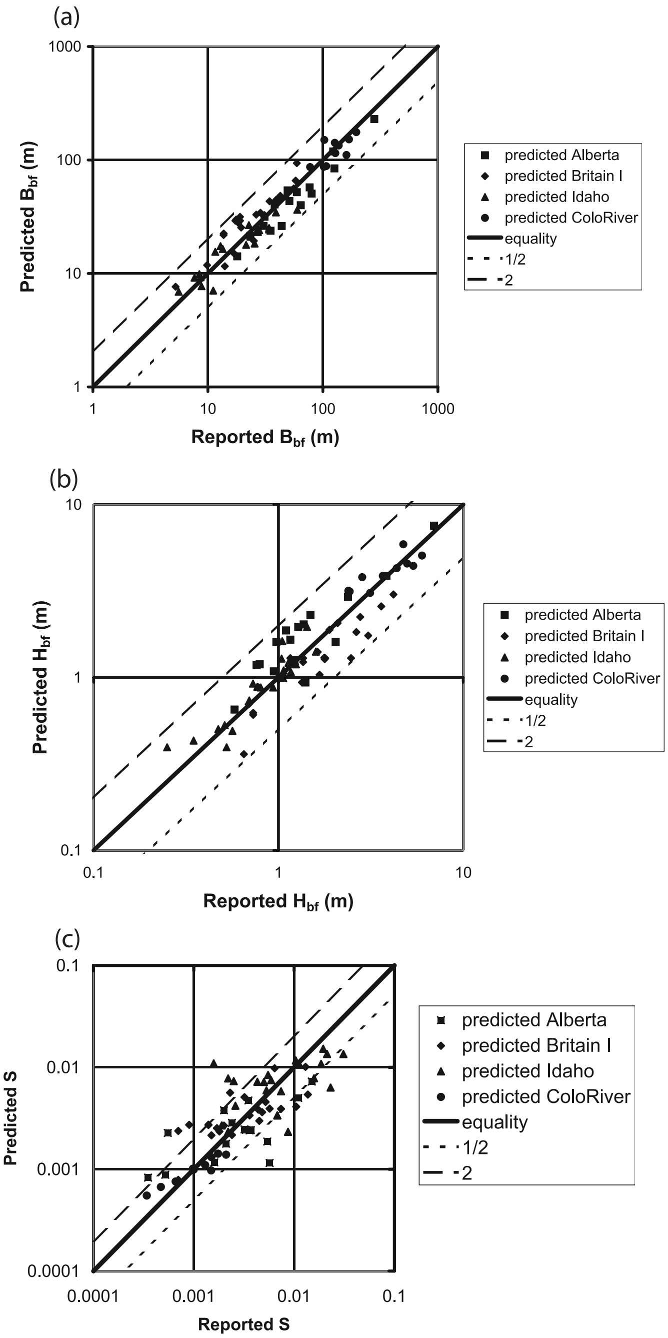

Figure 3. (a) Predicted versus reported bankfull width B_{bf} for the baseline data set. (b) Predicted versus reported bankfull depth H_{bf} for the baseline data set. (c) Predicted versus reported down-channel bed slope S for the baseline data set.

be due to measurement error, particularly in the measurement of D_{s50} and S. There is, however, another compelling reason for scatter in the slope relation. Mobile bed rivers are free to change their bankfull width and depth over short geomorphic time (e.g., hundreds or thousands of years). Slope changes other than those associated with changes in sinuosity, however, require a complete restructuring of the long profile of the river. Such a restructuring must occur over much longer geomorphic timescales, over which such factors as tectonism, climate change and sea level variation make themselves felt (and thus enter as "other parameters" (3)). This notwithstanding, the slope relation still shows a considerable degree of systematic variation.

[20] Both the predictive quality of the relations (10a), (11), and (10c) and the extent to which "other parameters" are felt can also be studied by plotting values of B_{bf}, H_{bf} and S predicted from (10a), (11), and (10c) versus the reported values. Figure 3a shows predicted versus observed values for B_{bf}. All of the 72 predicted values are between 1/2 and 2 times the reported values. Figure 3b shows predicted versus observed values for H_{bf}; again, all of the 72 predicted values are between 1/2 and 2 times the reported values.

Table 1. Average Values for $(X)_{pred}/(X)_{rep}$ for Seven Data Sets[a]

Average of Discrepancy Ratio	$(B_{bf})_{pred}/(B_{bf})_{rep}$	$(H_{bf})_{pred}/(H_{bf})_{rep}$	$(S)_{pred}/(S)_{rep}$
Alberta	0.83	1.27	1.16
Britain I	1.30	0.81	1.32
Idaho	0.97	1.08	1.38
ColoRiver	0.98	1.07	1.00
ColoSmall	1.06	1.10	0.87
Maryland	1.00	0.99	1.25
Britain II	1.34	0.91	0.99

[a]Here X = bankfull width B_{bf}, bankfull depth H_{bf}, and down-channel slope S.

Figure 3c shows predicted versus observed values of S; 52 of the 72 predicted values, or 72% are within 1/2 and 2 times the reported values.

[21] Variation within the data sets can be studied in terms of the average value of the ratio $(X)_{pred}/(X)_{rep}$ for each set, where $(X)_{pred}$ denotes the predicted value of parameter X and $(X)_{rep}$ denotes the reported value. These results are given in Table 1. As noted above, the Alberta streams are seen to be systematically wider and shallower, and the Britain I streams narrower and deeper, than that predicted by the regression relations. The average ratios $(B_{bf})_{pred}/(B_{bf})_{rep}$, $(H_{bf})_{pred}/(H_{bf})_{rep}$ and $(S)_{pred}/(S)_{rep}$ are nevertheless in all cases sufficiently close to unity to strengthen the case for quasi-universality of the relations.

5. Comparison of the Regression Relations Against Three Independent Sets of Data

[22] Three independent sets of data on gravel bed rivers are used to test the regression relations presented above. The first of these consists of 24 stream reaches from Colorado compiled by *Andrews* [1984], none of which includes the Colorado River itself. This set is referred to here as "ColoSmall." The ranges of parameters for the "ColoSmall" data mostly fall within the corresponding ranges of the baseline data set, but the former set does include some smaller streams. The second of these consists of 11 stream reaches from Maryland and Pennsylvania, USA [*McCandless*, 2003], here referred to as "Maryland" for short. The original data set contained 14 reaches, but three of these were excluded because (1) the stream was bedrock or (2) the value of D_{s50} was substantially below the range of the baseline set (27 mm to 167.5 mm) or c) the value of S was substantially above the range of the baseline set (0.00034 to 0.031). The third set of data is the British set of 62 reaches compiled by *Hey and Thorne* [1986]. The specific reaches in this set, which we here term "Britain II" for short, are largely different from those in the Britain I compilation of *Charlton et al.* [1978] used earlier to derive (10a), (11), and (10b).

[23] The ColoSmall, Maryland and Britain II data are plotted in Figure 4, which has the same format as Figure 1. The regression lines in Figure 4 are (10a), (11), and (10c), i.e., those determined using only the baseline data set. The ColoSmall and Maryland data sets show no systematic deviation from the regression lines determined from the baseline data set. The Britain II data set shows the same deviation as the Britain I data set; that is, the channels tend to be somewhat narrower and deeper.

[24] This systematic deviation is explored in more detail in Figures 5a, 5b, and 5c, where $(B_{bf})_{pred}$ is plotted against $(B_{bf})_{rep}$, $(H_{bf})_{pred}$ is plotted against $(H_{bf})_{rep}$ and $(S)_{pred}$ is plotted against $(S)_{rep}$, respectively. In Figure 5a all but 4 of the 97 predicted values of bankfull width for the ColoSmall, Maryland and Britain II sets are between 1/2 and 2 times the reported values. The 4 exceptions are all Britain II reaches, and in all 4 cases (10a) overpredicts the width.

[25] In Figure 5b all but 1 of the 97 predicted values of bankfull depth for the ColoSmall, Maryland and Britain II

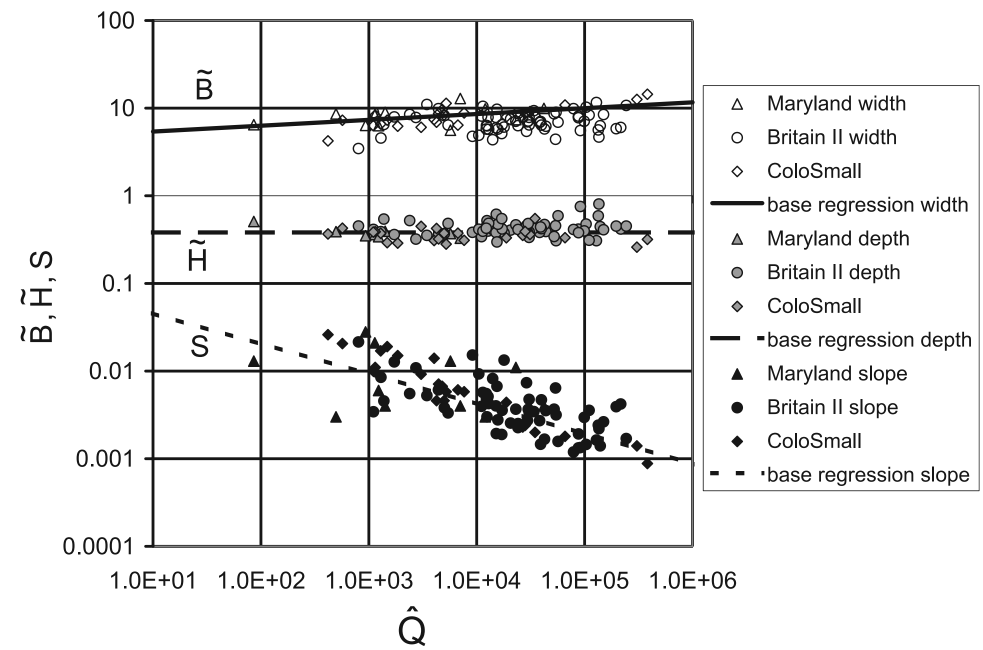

Figure 4. Dimensionless bankfull width $\tilde{B}$, dimensionless bankfull depth $\tilde{H}$, and down-channel bed slope S as functions of dimensionless bankfull discharge $\hat{Q}$ for the ColoSmall, Maryland, and Britain II data subsets, along with the power regression lines determined from the baseline data set.

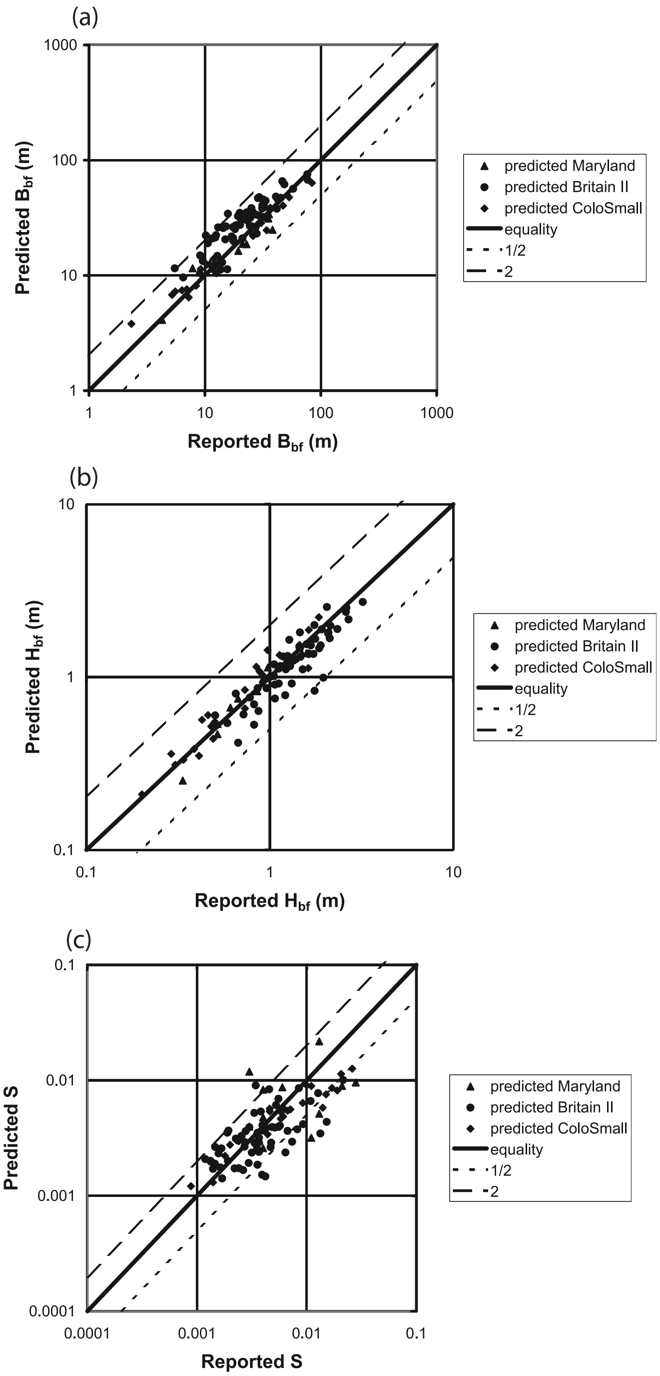

Figure 5. (a) Predicted versus reported bankfull width B_{bf} for the ColoSmall, Maryland, and Britain II data subsets. (b) Predicted versus reported bankfull depth H_{bf} for the ColoSmall, Maryland, and Britain II data subsets. (c) Predicted versus reported down-channel bed slope S for the ColoSmall, Maryland, and Britain II data subsets.

sets are between 1/2 and 2 times the reported values. The single exception is a Britain II reach, for which (11) underpredicts the depth.

[26] In Figure 5c 78 of the 97 predicted values for slope for the ColoSmall, Maryland and Britain II sets, or 80%, are within 1/2 and 2 of the reported values. Of the remaining 19 values, 3 are ColoSmall reaches, 6 are Maryland reaches and 10 are Britain II reaches; all but three of these values correspond to underpredictions of slope. Averages of the ratio of predicted to reported values for the ColoSmall, Maryland and Britain II sets are given in Table 1.

[27] A comparison of the values given in Table 1 allows the following initial conclusions. The ColoSmall, Maryland and Britain II data sets fit within the quasi-universal framework of the baseline data set. The Britain II data, however, show the same bias toward narrower, deeper channels as the Britain I set.

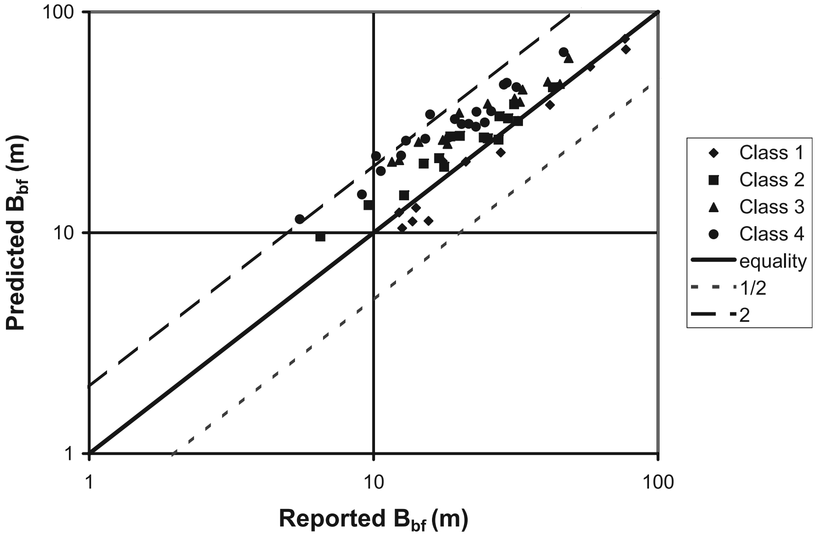

Figure 6. Predicted versus reported bankfull width B_{bf} for the Britain II data stratified according to vegetation density. Class 1 refers to the lowest, and Class 4 refers to the highest vegetation density.

[28] The largest deviation from universality is for the case of bankfull width of the Britain II streams, where B_{bf} is on the average overpredicted by (10a) by a factor of 1.34. Information from *Hey and Thorne* [1986] allows a quantification of this deviation. The authors have classified reaches of the data set on a scale from 1 to 4 in terms of the density of bank vegetation, with 1 denoting the lowest density. In Figure 6 the predicted and reported values of B_{bf} are given with the data discriminated according to vegetation density. Equation (10a) mildly underpredicts the bankfull width for the streams with the least dense bank vegetation, and noticeably overpredicts bankfull width for the streams with the densest bank vegetation The average of the discrepancy ratios $(B_{bf})_{pred}/(B_{bf})_{rep}$ for the four classes of vegetation are as follows: class 1, 0.93; class 2, 1.21; class 3, 1.45 and class 4, 1.66. As previously concluded by *Hey and Thorne* [1986] in regard to this data set, vegetation appears to exert a measurable control on bankfull width. In the present case this control is expressed as a deviation from universality in the dimensionless relation for bankfull width, with higher bank vegetation favoring narrower channels. The channels closest to universality are those with the lowest density of vegetation.

[29] The above observation concerning bank vegetation is broadly consistent with observations of vegetation effects on multithread channels reported by *Gran and Paola* [2001] and *Tal et al.* [2004]. A further step in the analysis would be to quantify the reduction in width with suitable measures of vegetal influence, including areal stem and root density, vegetation height, etc. A first important step in this direction has been made by *Millar* [2005], who has quantified the combined effects of bank vegetation and cohesive bank soil into a ratio of bank critical Shields number to bed critical Shields number.

6. Toward the Physics Underlying the Dimensionless Relations

[30] Equations (10a), (11), and (10c) presumably reflect the underlying physics of alluvial, single-thread gravel bed streams. It is thus useful to ask what physical assumptions would yield these same equations as a result. The analysis presented here is of necessity "broad brush," but is nevertheless intended to identify the factors controlling relations for hydraulic geometry.

[31] We begin by defining suitable parameters. Boundary shear stress at bankfull flow is denoted as $\tau_{b,bf}$, water density is denoted as ρ, sediment density is denoted as ρ_s volume gravel bed load transport rate at bankfull flow is denoted as $Q_{b,bf}$ and cross-sectionally averaged flow velocity is denoted as U_{bf}. Water conservation requires that

$$U_{bf} = \frac{Q_{bf}}{B_{bf} H_{bf}} \tag{15}$$

The normal flow approximation is used here to evaluate the boundary shear stress $\tau_{b,bf}$ and the shear velocity at bankfull flow $u_{*,bf}$:

$$\tau_{b,bf} = \rho g H_{bf} S \tag{16a}$$

$$u_{*,bf} = \sqrt{\frac{\tau_{b,bf}}{\rho}} = \sqrt{g H_{bf} S} \tag{16b}$$

The submerged specific gravity R of the gravel is defined as

$$R = \frac{\rho_s}{\rho} - 1 \tag{17}$$

For natural sediments R is usually close to the value of 1.65 for quartz. The Shields number τ^*_{bf} and Einstein number q^*_{bf}, both at bankfull flow and based on sediment size D_{s50}, are defined as

$$\tau^*_{bf} = \frac{\tau_{b,bf}}{\rho R g D_{s50}} \tag{18a}$$

$$q^*_{bf} = \frac{Q_{b,bf}}{B_{bf}\sqrt{R g D_{s50}}\ D_{s50}} \tag{18b}$$

In addition, a dimensionless bankfull gravel bed load transport rate $\hat{Q}_b$ analogous to the dimensionless water discharge $\hat{Q}$ is defined as

$$\hat{Q}_b = \frac{Q_{b,bf}}{\sqrt{gD_{s50}}\, D_{s50}^2} \quad (19)$$

[32] We assume that the relations that underlie (10a), (11) and (10b) involve (1) frictional resistance, (2) transport of gravel, (3) a channel-forming Shields number, (4) a relation for critical Shields number for the onset of gravel motion and (5) a relation for gravel "yield." (The reason for the quotes becomes apparent below.) Frictional resistance is described in terms of a relation of Manning-Strickler type:

$$\frac{U_{bf}}{u_{*,bf}} = \alpha_r \left(\frac{H_{bf}}{D_{s50}}\right)^{n_r} \quad (20a)$$

where the dimensionless parameters α_r and n_r are to be determined. Reducing with (15) and (16b),

$$\frac{Q_{bf}}{B_{bf} H_{bf} \sqrt{gH_{bf} S}} = \alpha_r \left(\frac{H_{bf}}{D_{s50}}\right)^{n_r} \quad (20b)$$

Gravel transport is described in terms of the *Parker* [1978] approximation of the *Einstein* [1950] relation applied to bankfull flow:

$$q_{bf}^* = \alpha_G \left(\tau_{bf}^*\right)^{3/2} \left(1 - \frac{\tau_c^*}{\tau_{bf}^*}\right)^{4.5} \quad (21)$$

where τ_c^* is a critical Shields number for the onset of motion and α_G is a coefficient equal to 11.2. Channel form is described in terms of a relation of the form

$$\tau_{bf}^* = r\tau_c^* \quad (22)$$

as described by *Parker* [1978], *Paola et al.* [1992], *Parker et al.* [1998], and *Dade and Friend* [1998]. As noted below, the parameter r provides a surrogate for bank strength. As such it is likely related to the parameter μ' used by *Millar* [2005] to characterize bank strength. Both *Millar* [2005] and *Knight et al.* [1994], emphasize the distinction between bed and bank shear stresses.

[33] Equation (21) reduces with (16a) and (18a) to

$$\frac{Q_{b,bf}}{\sqrt{gD_{s50}}\, D_{s50}^2} = \frac{\alpha_G}{R} \frac{B_{bf}}{D_{s50}} \left(\frac{H_{bf} S}{D_{s50}}\right)^{3/2} \left(1 - \frac{1}{r}\right)^{4.5} \quad (23)$$

In the *Parker* [1978] approximation of the *Einstein* [1950] bed load relation τ_c^* is taken to be a constant equal to 0.03. Here it is taken to be a (weak) function of $\hat{Q}$ such that the average value for the baseline data set is 0.03;

$$\tau_c^* = \alpha_\tau \hat{Q}^{n_\tau} \quad (24)$$

As will become apparent below, the above form is dictated by the forms of (10a), (11), and (10c) and the framework of the present analysis. Between (5c), (16a), (18a), and (22) we find that (24) reduces to

$$\frac{H_{bf} S}{RD_{s50}} = r\alpha_\tau \left(\frac{Q_{bf}}{\sqrt{gD_{s50}}\, D_{s50}^2}\right)^{n_\tau} \quad (25)$$

Finally, a gravel "yield" relation describes how the gravel bed load transport rate at bankfull flow $Q_{b,bf}$ varies with bankfull flow Q_{bf} and grain size D_{s50};

$$\hat{Q}_b = \alpha_y \hat{Q}^{n_y} \quad (26a)$$

where α_y and n_y are dimensionless parameters that we compute below. Reducing (26a) with (5a) and (19),

$$\frac{Q_{b,bf}}{\sqrt{gD_{s50}}\, D_{s50}^2} = \alpha_y \left(\frac{Q_{bf}}{\sqrt{gD_{s50}}\, D_{s50}^2}\right)^{n_y} \quad (26b)$$

Between (23) and (26b),

$$\frac{\alpha_G}{R} \frac{B_{bf}}{D_{s50}} \left(\frac{H_{bf} S}{D_{s50}}\right)^{3/2} \left(1 - \frac{1}{r}\right)^{4.5} = \alpha_y \left(\frac{Q_{bf}}{\sqrt{gD_{s50}}\, D_{s50}^2}\right)^{n_y} \quad (27)$$

[34] The above relations contain the unevaluated dimensionless coefficients α_r, α_τ and α_y and exponents n_r, n_τ and n_y. We now compute these parameters so as to yield precisely the coefficients α_B and α_S, exponents n_B and n_S and the constant $\tilde{H}_o$ determined by regression from the baseline data set, i.e., the values given in (10a), (10c), and (11). Before completing this step, however, some elaboration of the above relations is appropriate.

[35] Equation (20a) is a Manning-Strickler relation of the general form that *Bray* [1979] and *Parker* [1991] have applied to gravel rivers; it is also similar to related logarithmic forms for gravel bed rivers [e.g., *Limerinos*, 1970; *Hey*, 1979; *Bray*, 1979]. As such, it is appropriate for a broad brush formulation. There are two reasons why it cannot be accurate in detail. The first of these is the fact that the characteristic grain size on which grain roughness (skin friction) depends is a size coarser than D_{s50}; commonly used sizes are D_{s90} and D_{s84}. The second of these is the likelihood that not all the drag in gravel bed rivers at bankfull flow is due to skin friction. Bar structures, planform variation and bank vegetation can give rise to at least some form drag [e.g., *Millar*, 1999]. The issue of form drag is discussed in more detail below.

[36] The *Parker* [1978] approximation of the *Einstein* [1950] bed load transport relation embodied in (21) is also an appropriate broad brush relation for gravel bed rivers. There are at least three reasons why it cannot be accurate in detail: (1) it does not account for gravel mixtures [e.g., *Parker*, 1990; *Wilcock and Crowe*, 2003], (2) no attempt has been made to remove the effect of form drag (which would reduce the total bed load transport rate), and (3) no attempt has been made to account for preferential "patches" or "lanes" (which would increase the total transport rate [*Paola and Seal*, 1995]).

[37] The original derivation of the relation for channel form (22) presented by *Parker* [1978] does not account for the effect of form drag or planform variation, both effects

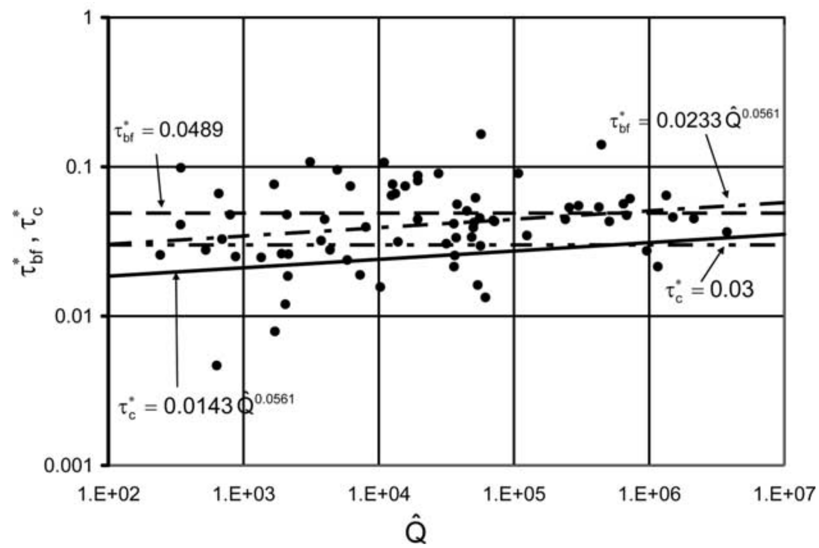

Figure 7. Plot of the bankfull Shields number τ^*_{bf} for the baseline data set. Also included are the line τ^*_{bf} = 0.0489 corresponding to the average value for the baseline data set, relation (38) for τ^*_{bf}, the estimate of critical Shields number τ^*_c = 0.03, and relation (36) for critical Shields number.

that are felt here. This notwithstanding, *Paola et al.* [1992] and *Parker et al.* [1998] have shown its value as a broad brush relation.

[38] According to (24) the critical Shields number τ^*_c at the onset of motion depends on dimensionless discharge $\hat{Q}$. In the original *Parker* [1978] approximation of the *Einstein* [1950] bed load transport relation τ^*_c is a constant equal to 0.03. We demonstrate below, however, that the exponent n_τ in (24) is very small.

[39] Finally, the gravel "yield" relation (26a) does not involve mean annual gravel yield, but rather the gravel transport rate at bankfull flow. One presumably scales with the other, but the details of the scaling have yet to be worked out. The "yield" relation relates to processes at the scale of the drainage basin rather than local in-channel processes. More specifically, it implies that catchments organize themselves to provide gravel during floods such that the gravel discharge scales as a power law of the water discharge. Equation (26a) is the most empirical of the relations used here.

[40] Substituting (10a), (11), and (10c) into (20b), (25), and (27) results in the evaluations

$$\alpha_r = \alpha_B^{-1}\alpha_S^{-1/2}\tilde{H}_o^{-[(3/2)+(5/4)n_S-(5/2)n_B]} \tag{28a}$$

$$n_r = \frac{5}{2}\left(\frac{1}{2}n_S - n_B\right) \tag{28b}$$

$$\alpha_\tau = \frac{\tilde{H}_o\alpha_S}{rR} \tag{29a}$$

$$n_\tau = \frac{2}{5} - n_s \tag{29b}$$

$$\alpha_y = \frac{\alpha_G\left(1-\frac{1}{r}\right)^{4.5}\alpha_B\tilde{H}_o^{3/2}\alpha_S^{3/2}}{R} \tag{30a}$$

$$n_y = 1 + n_B - \frac{3}{2}n_S \tag{30b}$$

The parameter r is evaluated as follows. Figure 7 shows a plot of $\tau^*_{\rm bf}$ as computed from (16a) and (18a), i.e.,

$$\tau^*_{bf} = \frac{H_{bf}S}{RD_{s50}} \tag{31}$$

versus $\hat{Q}$ for the baseline data set. The average value $\langle\tau^*_{\rm bf}\rangle$ for the baseline data set is found to be

$$\langle\tau^*_{bf}\rangle = 0.0489 \tag{32}$$

Using (22) and the original estimate of τ^*_c of 0.03 in the *Parker* [1978] approximation of the *Einstein* [1950] bed load transport relation, we obtain the following estimate for r:

$$r = 1.63 \tag{33}$$

[41] Substitution of (10a), (10c), (11) and (33) into (28), (29) and (30) yields the values for α_r, α_τ, α_Y, n_r, n_τ and n_y:

$$\alpha_r = 3.71 \tag{34a}$$

$$\alpha_\tau = 0.0143 \tag{34b}$$

$$\alpha_y = 0.00330 \tag{34c}$$

$$n_r = 0.263 \tag{34d}$$

$$n_\tau = 0.0561 \tag{34e}$$

$$n_y = 0.551 \tag{34f}$$

and thus the following evaluations for (20a), (24) and (26):

$$\frac{U_{bf}}{u_{*,bf}} = 3.71\left(\frac{H_{bf}}{D_{s50}}\right)^{0.263} \tag{35}$$

$$\tau_c^* = 0.0143 \quad \hat{Q}^{0.0561} \tag{36}$$

$$\hat{Q}_b = 0.00330 \quad \hat{Q}^{0.551} \tag{37}$$

In addition, between (22), (33) and (36) it is found that

$$\tau_{bf}^* = 0.0233 \quad \hat{Q}^{0.0561} \tag{38}$$

[42] The exponent in the resistance relation (35) of 0.263 is somewhat larger than the standard Manning-Strickler exponent of $1/6 \cong 0.167$. Relations (38) for bankfull Shields number and (37) for critical Shields number show a very weak dependence on $\hat{Q}$. This weak dependence is reflected in the baseline data set: a direct regression of the data of Figure 7 yields a nearly identical relation with a coefficient of 0.0230 and an exponent of 0.0572. The exponent is significantly different from zero only at the 90% level, but not at 95%; p = 0.078. This notwithstanding, (36) represents an empirical improvement over a constant critical Shields number of 0.03, for the following reason. Most alluvial gravel bed rivers can be expected to be competent to move their median surface size D_{s50} at bankfull flow [e.g., *Andrews*, 1983; *Hey and Thorne*, 1986]. In the case of a constant critical Shields number of 0.03, 21 of 72 reaches in Figure 7, or 29% plot below the threshold of motion at bankfull flow, whereas in the case of (36) only 12 reaches, or 17% plot below the threshold of motion. This empirically derived weak dependence of τ_c^* on $\hat{Q}$ may represent a consequence of form drag.

[43] The exponent in the gravel "yield" relation of (37) indicates that the gravel transport rate at bankfull flow should increase as about the square root of the bankfull discharge. Thus the volume concentration of transported gravel should decline downstream. Since water discharge usually increases nearly linearly with contributing drainage area, the implication is that "gravel yield" increases with contributing area at a rate that is markedly slower than linear, i.e., roughly as the 0.5 power of contributing area. The explanation and implications of this inference remain to be explored in future work. Irrespective of its origin, (37) likely expresses a property of how drainage basins organize themselves, rather than local properties in the channel. It is likely, however, that as down-channel slope S drops with increasing water discharge in accordance with (10c), the adjacent hillslopes often become less steep, so delivering less sediment (and thus less gravel) for the same unit rainfall. This reduced gravel delivery is likely mitigated by downstream fining of the gravel itself.

7. Quantification of Deviation From Universality

[44] The derivation of the physical relations underlying hydraulic geometry allows for a quantification of deviations from universality. This further allows for a characterization of the effect of the "other" parameters in (3a) $\sim$ (3c). In order to do this, the physical relations of the previous section are adopted as primary. The derivation leading to (28) $\sim$ (30) is then inverted so that the coefficients and exponents in the dimensionless relations for hydraulic geometry become functions of the parameter r, and coefficients α_r, α_τ and α_y and the exponents n_r, n_τ and n_y of the physical relations. This yields the following coefficients and exponents describing generalized power relations for hydraulic geometry:

$$\alpha_B = \frac{\alpha_y}{\sqrt{R}\quad \alpha_G\left(1-\frac{1}{r}\right)^{4.5}(r\alpha_\tau)^{3/2}} \tag{39a}$$

$$n_B = \frac{1}{5} - \frac{1}{2}n_\tau - \frac{2}{5}n_r \tag{39b}$$

$$\tilde{H}_o = \left[\frac{\alpha_G\left(1-\frac{1}{r}\right)^{4.5} r\alpha_\tau}{\alpha_y\alpha_r}\right]^{\frac{1}{1+n_R}} \tag{40}$$

$$\alpha_S = R\alpha_\tau\left[\frac{\alpha_G\left(1-\frac{1}{r}\right)^{4.5} r\alpha_\tau}{\alpha_y\alpha_r}\right]^{-\left(\frac{1}{1+n_R}\right)} \tag{41a}$$

$$n_S = \frac{2}{5} - n_\tau \tag{41b}$$

[45] Here we examine the effect of variation of the following parameters on the deviation from universality: r, α_r and α_y. This deviation is expressed in terms of the parameters α_B, $\tilde{H}_o$ and α_S as specified by (39a), (40) and (41a), respectively. The parameter r, i.e., the ratio of bankfull Shields number to critical Shields number, can be thought of as a measure of "bank strength," in that channels with stronger banks can maintain higher values of τ_{bf}^* relative to τ_c^* [see also *Millar*, 2005]. Using information from *Rice* [1979] and *Ashmore* [1979], *Parker* [1982] deduced a mean value of τ_{bf}^* of 0.0420, and thus a value of r of about 1.4 for anabranches of the braided gravel bed Sunwapta River, Jasper National Park, Canada, which flows on an unvegetated valley flat. This value represents a lower limit in the absence of vegetation and cohesive sediment to add bank strength. The average value of r of 1.63 deduced

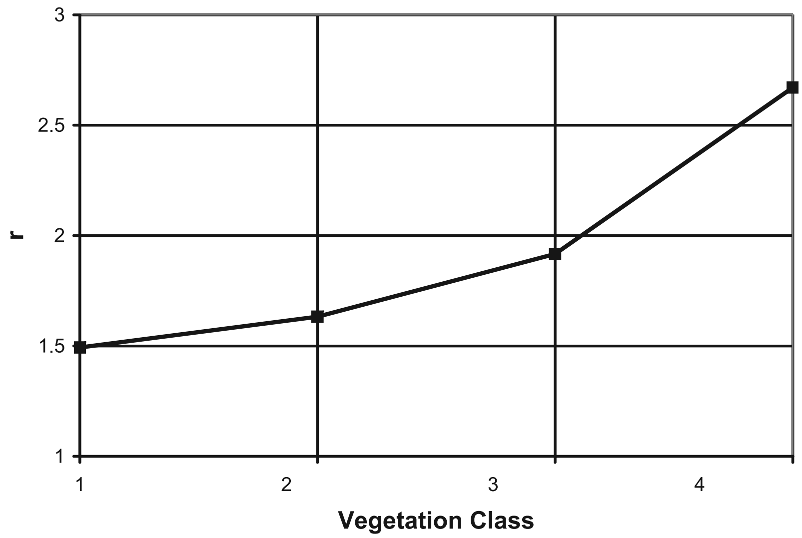

Figure 8. Plot of the parameter r estimating the ratio of bankfull Shields number to critical Shields number as a function of vegetation density for the Britain II data. Class 1 refers to the lowest, and Class 4 refers to the highest vegetation density.

for the baseline data set presented here is rather higher. The Britain II data can be used to provide a qualitative measure of the effect of bank vegetation density on r. Figure 8 shows a plot of the average value of r for each vegetation density class of the Britain II data. Here r is calculated in the same way as for the baseline data, i.e., from (31), (22), and an estimated value of τ_c^* of 0.03. The parameter r takes the following values in order of vegetation density: 1.49 (class 1, lowest vegetation density); 1.63 (class 2), 1.92 (class 3) and 2.67 (class 4, highest vegetation density). For reference, the value of r determined from the baseline data set is 1.63. In testing the effect of varied r on the predicted values of α_B, $\tilde{H}_o$ and α_S, r is allowed here to vary from 0.9 to 1.1 times the baseline value of 1.63.

[46] Channel resistance decreases as the parameter α_r in the Manning-Strickler relation (20a) increases. This can be seen by defining a dimensionless resistance coefficient C_f as

$$C_f = \frac{\tau_{b,bf}}{\rho U_{bf}^2} \tag{42}$$

Between (15), (16a), (20b), and (42) we find that

$$C_f = \alpha_r^{-2} \left(\frac{H_{bf}}{D_{s50}} \right)^{-2n_r} \tag{43}$$

Here α_r is allowed to vary from 0.8 to 1.2 times its baseline value of 3.71, and the associated values of α_B, $\tilde{H}_o$ and α_S are predicted accordingly. At the lower value the resistance coefficient C_f is increased by a factor of 1.56; at the higher value C_f is decreased by a factor of 0.69.

[47] Gravel supply increases linearly with increasing parameter α_y in the "gravel yield" relation (26a). Here α_y is allowed to vary from 0.5 to 1.5 times its baseline value of 0.00330, and the associated values of α_B, $\tilde{H}_o$ and α_S are predicted accordingly.

[48] The effects of the variation of r, α_r and α_y on coefficients α_B and α_S in (39a) and (41a), respectively, and the parameter $\tilde{H}_o$ in (40) are summarized in Table 2 and Figures 9a, 9b, and 9c. The effect of varying r is illustrated in Figure 9a. Increasing r (i.e., increasing "bank strength") from 0.9 to 1.1 times the baseline value results in an bankfull channel that is increasingly narrower and has an increasingly lower bed slope. A comparison with the data in Figure 9a suggests that bank strength is one reason why the Alberta reaches are wider and shallower than the Britain I reaches.

[49] The effect of varying α_r is studied in Figure 9b. Decreasing α_r from 1.2 to 0.8 times the baseline value, and thus increasing the channel resistance coefficient from 0.69 to 1.56 times that which would be predicted using the baseline value of α_y, results in a bankfull channel that is increasingly deep and has an increasingly lower slope. Changing α_r has no effect on channel width.

[50] The effect of varying α_y is shown in Figure 9c. Increasing α_y (and thus gravel supply) from 0.5 to 1.5 times the baseline value results in a bankfull channel that is increasingly wider, shallower and steeper. A comparison

Table 2. Effect of Variation of the Parameters r, α_r, and α_y on the Parameters $\tilde{H}_o$, α_B, and α_y

r	r Factor	$\tilde{H}_o$	α_B	α_S
1.79	1.1	0.696	2.19	0.0578
1.63	1	0.400	4.63	0.101
1.47	0.9	0.184	12.97	0.218
α_y	α_y Factor	$\tilde{H}_o$	α_B	α_S
0.00531	1.5	0.290	6.95	0.139
0.00354	1	0.400	4.63	0.101
0.00177	0.5	0.692	2.32	0.0581
α_r	α_r Factor	$\tilde{H}_o$	α_B	α_S
4.11	1.2	0.346	4.63	0.134
3.43	1	0.400	4.63	0.1001
2.74	0.8	0.477	4.63	0.0707

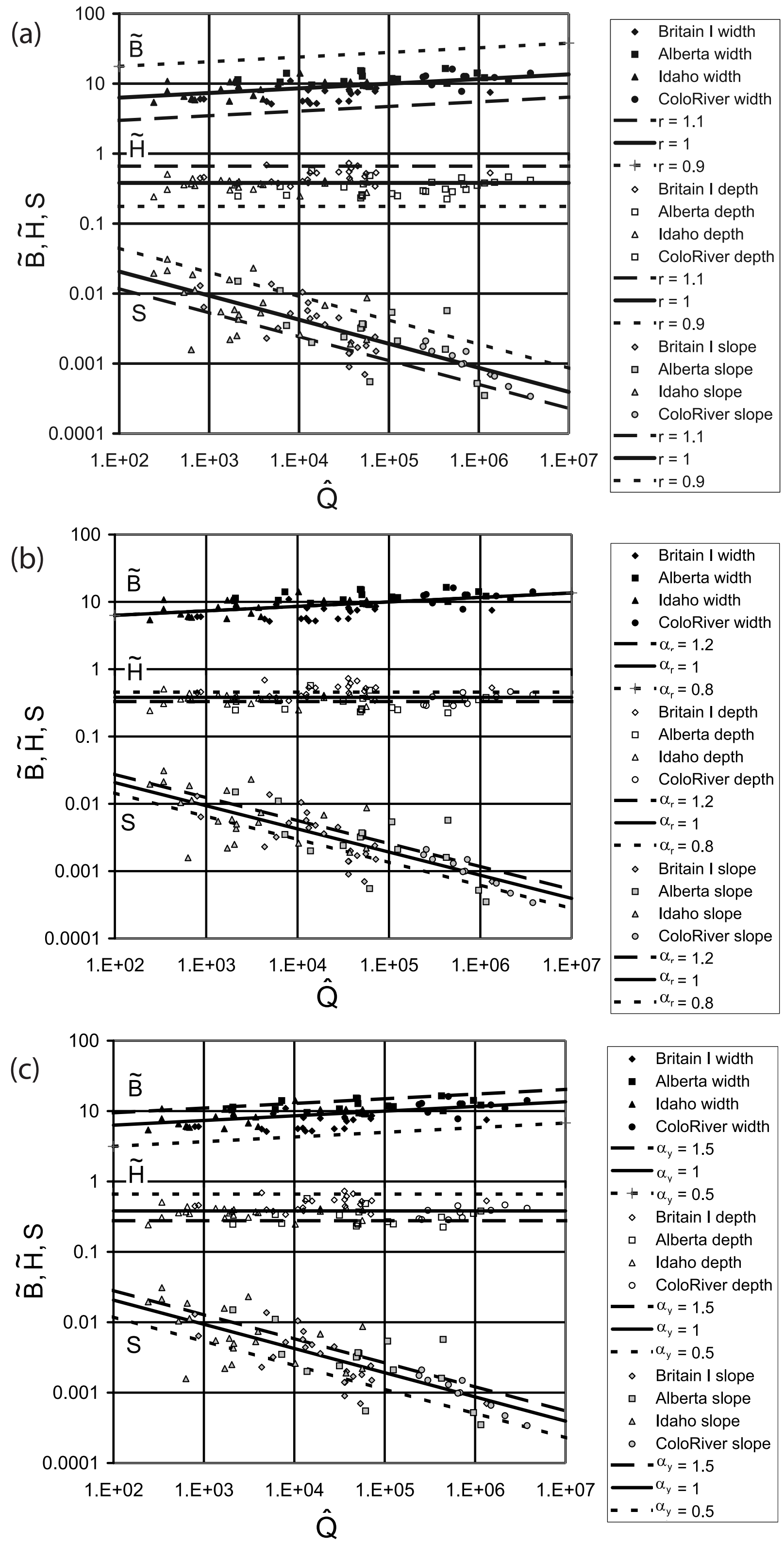
(a)
(b)
(c)
B̃
H̃
S
B̃, H̃, S
Q̂
100
10
1
0.1
0.01
0.001
0.0001
1.E+02
1.E+03
1.E+04
1.E+05
1.E+06
1.E+07
Britain I width
Alberta width
Idaho width
ColoRiver width
r = 1.1
r = 1
r = 0.9
Britain I depth
Alberta depth
Idaho depth
ColoRiver depth
Britain I slope
Alberta slope
Idaho slope
ColoRiver slope
α_r = 1.2
α_r = 1
α_r = 0.8
α_y = 1.5
α_y = 1
α_y = 0.5

Figure 9

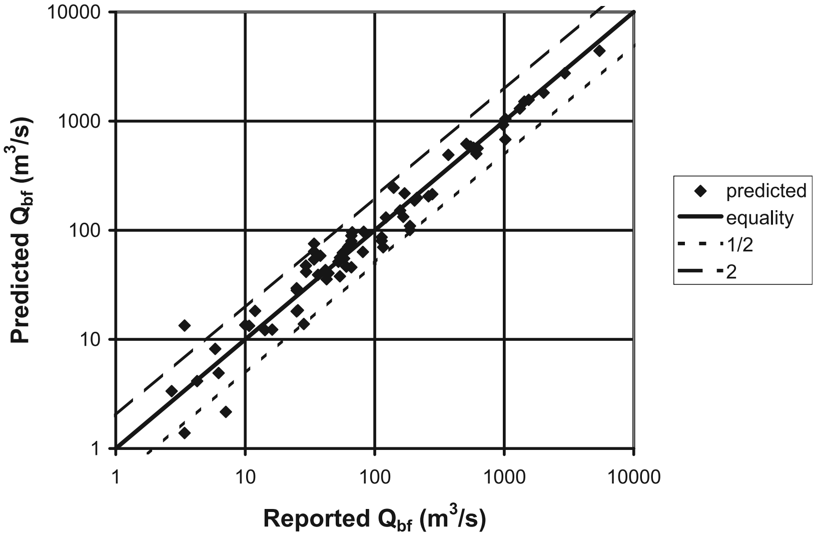

Figure 10. Predicted versus reported bankfull discharge for the baseline data set, discriminated according to subset.

with the data in Figure 9c suggests that another reason why the Alberta streams are wider and shallower than the Britain I streams may be that they have a higher gravel supply.

8. Predictor for Bankfull Discharge

[51] In general bankfull discharge should be determined from a rating curve of discharge versus stage, or by other direct methods [*Navratil et al.*, 2004; *Knight*, 2005]. In practice, however, such information is often not available.

[52] Equation (35) reduced with (16b) yields the following relation:

$$\frac{U_{bf}}{u_{*,bf}} = \frac{Q_{bf}}{B_{bf}H_{bf}\sqrt{gH_{bf}S}} = 3.71\left(\frac{H_{bf}}{D_{s50}}\right)^{0.263} \tag{44}$$

This relation provides a means for estimating bankfull discharge Q_{bf} from measured channel parameters B_{bf}, H_{bf}, S and D_{s50}. In Figure 10 the values of Q_{bf} predicted from (44) are compared against the measured values for the four baseline data sets used to derive (44). As expected, (44) passes through the middle of the data set from which it was determined by regression. Of more interest is the scatter. We find that 93% of the predicted values are seen to be between 1/2 and 2 times the reported values.

[53] The scatter in the data of Figure 10 is very small for measured discharges above 500 m^3/s. Most of these points refer to the Colorado River. The values for bankfull discharge for the ten reaches of the Colorado River are characteristic values determined with the use of a form of Manning's relation calibrated site specifically to the field data [*Pitlick and Cress*, 2000]. Evidently this procedure has reduced the scatter.

[54] An independent test of (44) is given in Figure 11 using the ColoSmall, Maryland, Britain II data sets. All 97 predicted values are seen to be between 1/2 and 2 times the reported values.

[55] The coefficient and exponent of (44) were back calculated from the dimensionless relations for hydraulic geometry. A direct regression using the baseline data set yields a very similar result:

$$\frac{U_{bf}}{u_{*,bf}} = 4.39\left(\frac{H_{bf}}{D_{s50}}\right)^{0.210} \tag{45}$$

Both these relations are in turn similar to an earlier one from *Bray* [1979], which is based on a subset of the data used here (Alberta):

$$\frac{U_{bf}}{u_{*,bf}} = 3.85\left(\frac{H_{bf}}{D_{s50}}\right)^{0.281} \tag{46}$$

Figure 9. (a) Dimensionless bankfull width $\tilde{B}$, dimensionless bankfull depth $\tilde{H}$, and down-channel bed slope S as functions of dimensionless bankfull discharge $\hat{Q}$, showing the predictions of the generalized hydraulic geometry relations as the parameter r is varied from 0.9 to 1.1. Increasing r is associated with increasing "bank strength." Also shown is the baseline data set discriminated according to subset. (b) Dimensionless bankfull width $\tilde{B}$, dimensionless bankfull depth $\tilde{H}$, and down-channel bed slope S as functions of dimensionless bankfull discharge $\hat{Q}$, showing the predictions of the generalized hydraulic geometry relations as the parameter α_r is varied from 0.8 to 1.2. Increasing α_r is associated with decreasing channel resistance. Also shown is the baseline data set discriminated according to subset. (c) Dimensionless bankfull width $\tilde{B}$, dimensionless bankfull depth $\tilde{H}$, and down-channel bed slope S as functions of dimensionless bankfull discharge $\hat{Q}$, showing the predictions of the generalized hydraulic geometry relations as the parameter α_y is varied from 0.5 to 1.5. Increasing α_y is associated with increasing gravel supply. Also shown is the baseline data set discriminated according to subset.

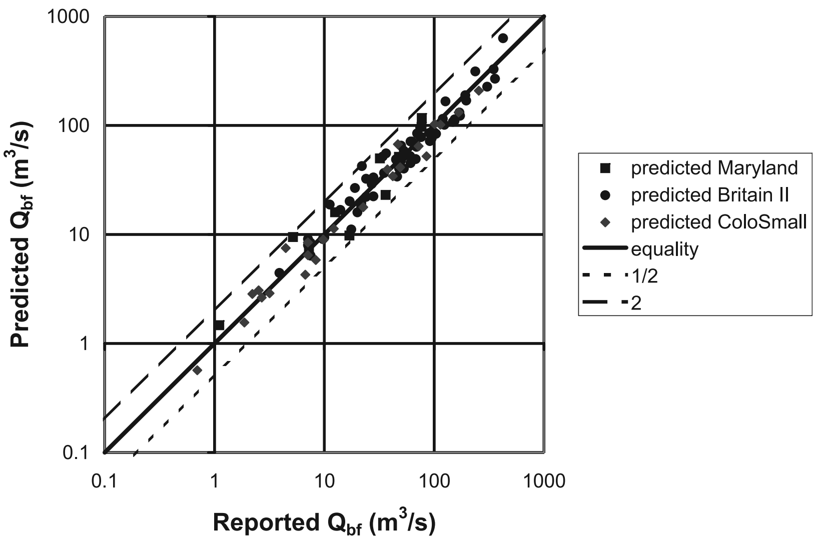

Figure 11. Predicted versus reported bankfull discharge for the Maryland, Britain II, and ColoSmall data sets.

All three relations are shown in Figure 12; (44) is the only one of them that is specifically derived from the hydraulic relations (10a), (11), and (10c).

9. Form Drag

[56] The resistance to flow in a river can be partitioned into skin friction, i.e., that part of the drag that acts directly on the grains themselves, and form drag, i.e., that part associated with bed forms such as bars, channel planform irregularities, etc. *Parker and Peterson* [1980] have argued that form drag in gravel bed streams is significant at low flow, but may be neglected at the flood flows that move gravel because the bars are effectively drowned. *Millar* [1999], on the other hand, has argued that form drag may be measurable at flood flows as well. The present analysis provides a basis for quantifying the partition between skin friction and form drag in gravel bed streams.

[57] An appropriate relation for the resistance coefficient C_{fs} due to skin friction alone (here applied to bankfull conditions) is

$$C_{fs}^{-1/2} = 8.1\left(\frac{H_{bf}}{k_s}\right)^{1/6} \tag{47}$$

where H denotes flow depth and k_s is a roughness height given as

$$k_s = 2\ \ D_{s90} \tag{48}$$

and D_{s90} is the surface size such that 90 percent is finer [*Parker*, 1991; *Wong*, 2003; *Wong and Parker*, 2006]. Total channel resistance is estimated with (44), which reduces with (16b) and (42) to the form

$$C_f^{-1/2} = 3.71\left(\frac{H_{bf}}{D_{s50}}\right)^{0.263} \tag{49}$$

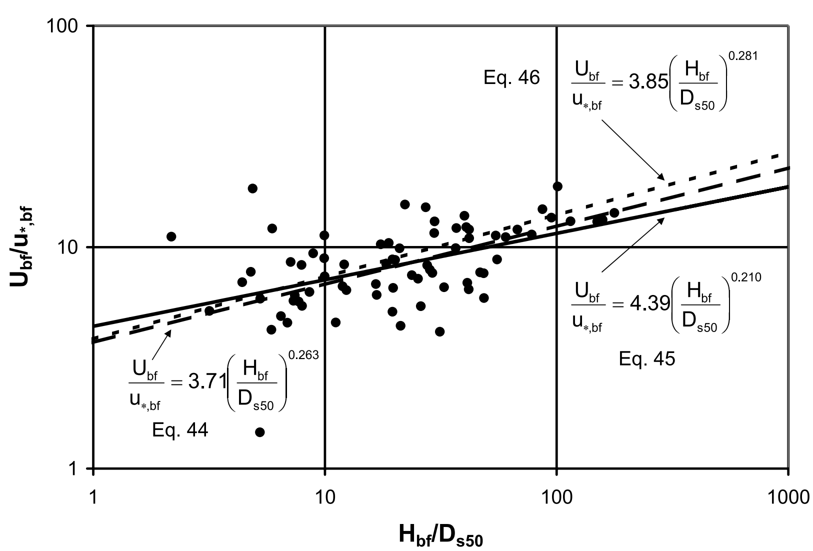

Figure 12. Plot of three forms for the Manning-Strickler resistance relation: (44), (45), and (46).

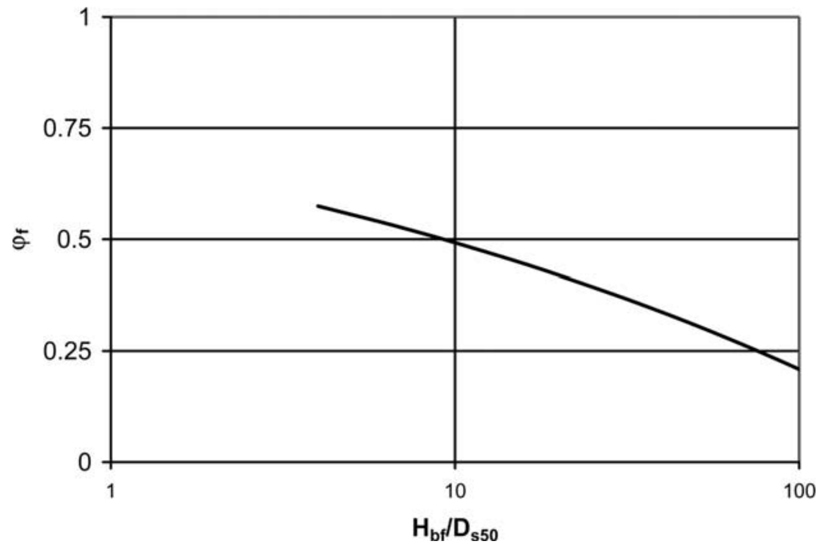

Figure 13. Estimated fraction φ_f of the resistance coefficient that is form drag versus the ratio H_{bf}/D_{s50}, based on the assumption that D_{s90}/D_{s50} is equal to 3.

The fraction of resistance φ_f due to form drag at bankfull flow is then given by the relation

$$\varphi_f = \frac{C_f - C_{fs}}{C_f} \tag{50}$$

where C_f is evaluated from (49) and C_{fs} is evaluated from (47) and the baseline values for α_r and n_r.

[58] The above relations allow for a specification of φ_f as a function of H_{bf}/D_{s50} upon specification of the ratio D_{s90}/D_{s50}. This parameter is a function of, among other things, sediment supply. *Mueller et al.* [2005] report values of both D_{s50} and D_{s90} for 32 gravel bed reaches in Idaho extracted from the compendium of *King et al.* [2004]. Many of the stream reaches in this set overlap with those in the Idaho data of *Parker et al.* [2003] used as baseline data here. The values of D_{s90}/D_{s50} in the data set of *Mueller et al.* [2005] ranges from a low value of 1.69 to a high value of 13.8, with a median value of 2.99. With this in mind the value D_{s90}/D_{s50} = 3 is used as an example. The resulting prediction for form drag is shown in Figure 13. The fraction of resistance that is form drag is predicted to decrease from 0.57 to 0.21 as H_{bf}/D_{s50} increases from 4 to 100, a range that captures the great majority of the reaches studied here. A refinement of the broad brush analysis presented above would involve removing this form drag in the calculation of gravel transport.

[59] Equation (38) indicates that the Shields number at bankfull flow τ^*_{bf} is a weak function of dimensionless discharge $\hat{Q}$ and nothing else. *Mueller et al.* [2005], however,

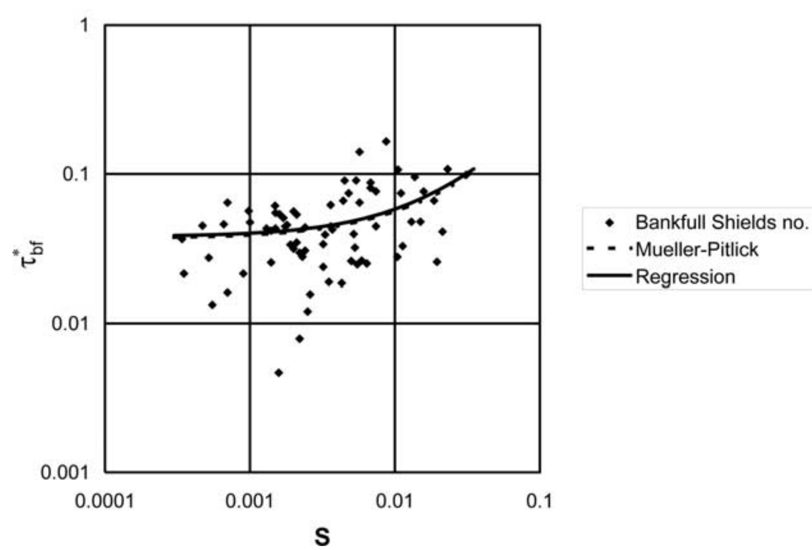

Figure 14. Plot of bankfull Shields number τ^*_{bf} versus bed slope S for the baseline data set. Also included is relation (51) [*Mueller et al.*, 2005] and the linear regression relation (52) obtained from the baseline data set.

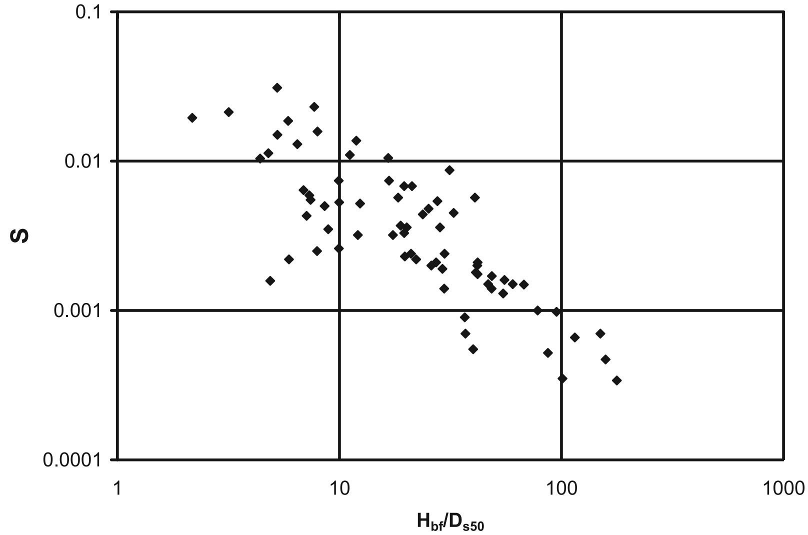

Figure 15. Plot of S versus H_{bf}/D_{s50} for the baseline data set.

have shown a tendency for τ^*_{bf} to increase with bed slope S as well. They applied linear regression to their data set to obtain the trend

$$\tau^*_{bf} = 1.91 \quad S + 0.037 \tag{51}$$

A plot of τ^*_{bf} versus S using the four baseline data sets of this paper is shown in Figure 14. While the scatter is considerable, the tendency for τ^*_{bf} to increase with increasing bed slope S is clear. A linear regression applied to the same baseline data results in the relation

$$\tau^*_{bf} = 2.00 \quad S + 0.038 \tag{52}$$

with a value r^2 associated with least squares regression of 0.165. As noted above, most of the stream reaches in the data set used by *Mueller et al.* [2005] overlap with those in the Idaho baseline data set used above. On the other hand, some 68 percent of the reaches in the baseline set (Alberta, Britain I and ColoRiver) do not overlap with those used by *Mueller et al.* [2005]. The good correspondence between (51) and (52) in Figure 14 thus suggests that the trend is real.

[60] *Mueller et al.* [2005] have speculated on the reasons why τ^*_{bf} tends to increases with increasing bed slope. One contributor to this effect might be form drag. Figure 13 suggests that the fraction of resistance that is form drag increases with decreasing values of H_{bf}/D_{s50}. Figure 15 illustrates that for the baseline data used here H_{bf}/D_{s50} correlates negatively with bed slope S. The implication is that form drag increases with increasing slope. If the bankfull Shields number associated with skin friction alone remains insensitive to slope, the total bankfull Shields number (including skin friction and form drag) should increase with increasing slope.

10. Discussion

[61] It is the dimensionless formulation used here that allows backing out the physics behind the relations for hydraulic geometry. This underlying physics in turn allows the study of, for instance, the dependence of hydraulic geometry on sediment supply or a measure of bank strength Such information cannot be easily extracted from dimensionally inhomogeneous equations obtained by means of regression applied directly to parameters of differing dimensions.

[62] Equation (37) indicates that the gravel transport rate at bankfull flow $Q_{b,bf}$ increases with bankfull discharge Q_{bf} to about the half power. *Mueller and Pitlick* [2005], however, have estimated a linear relation between annual gravel yield and bankfull flow for the Halfmoon Creek basin, a headwater catchment in Colorado. The reason for the discrepancy is not known at this time. It may be, however, that a decrease in the ratio of gravel yield to bankfull discharge would be realized if the analysis of *Mueller and Pitlick* [2005] were carried farther downstream into regions of lower bed slope. The discrepancy highlights the fact that the relations derived here apply as overall averages, and thus may be at variance with site-specific data.

[63] It should be emphasized that the regression relations proposed here should not be applied outside their range of applicability. For example, *Montgomery and Buffington* [1997] indicate that a step-pool topography may be expected for slopes S in excess of 0.03. The highest slope in either the baseline data set or the data set used to test the regression relations is 0.031. The present analysis does not apply to step-pool topography.

[64] The present broad brush theory accounts for neither armoring nor downstream fining, both of which are known to be important aspects of gravel bed streams. The theory could be extended to include these elements. The resulting formulation would not allow solution in closed form, and in particular in terms of power laws. It would likely, however, have improved predictive capacity.

[65] The approach to the physics underlying relations for hydraulic geometry of gravel bed rivers offered here stands in contrast to extremal formulations offered by, e.g., *Chang* [1980], *Yang et al.* [1981], *Huang et al.* [2002, 2004], *Eaton and Millar* [2004], and *Millar* [2005]. It has been known for

some time that specification of channel-based relations for flow resistance, gravel transport and channel form alone are insufficient to derive both the coefficients and exponents governing hydraulic geometry [e.g., *Parker*, 1979]. More specifically, one more constraint is required. One way to obtain this constraint is to apply an extremal condition that applies to flow and/or sediment transport conditions at the cross section itself. It has been variously proposed that channels adjust their cross sections to (1) minimize variance, (2) minimize unit stream power, (3) minimize total stream power, (4) maximize the friction coefficient, (5) maximize the sediment transport rate or efficiency and (6) minimize the Froude number. Surveys of these proposed constraints are given by *Soar and Thorne* [2001] and *Millar* [2005].

[66] Such approaches have met with some success in explaining hydraulic geometry [*Huang et al.*, 2002; *Millar*, 2005]. They nevertheless suffer from the drawback that the extremal condition in question must be accepted a priori. Here we offer a complete (albeit broad brush) formulation for the problem of hydraulic geometry that neither invokes nor requires any extremal condition. Reasonable, testable and dimensionally consistent relations for (1) hydraulic resistance, (2) gravel transport, (3) the threshold of motion and (4) channel-forming condition combined with a gravel "yield" relation of the form of (37) result in precisely the observed dimensionally consistent relations for hydraulic geometry obtained from the baseline data set.

[67] The difference between the two approaches is not trivial. Existing extremal formulations seek the extra condition by imposing it at a given cross section. In the present approach the extra constraint is a property of the drainage basin upstream of the cross section, i.e., the gravel "yield" relation.

[68] The actual long-term gravel "yield" relation is likely to be different from basin to basin depending upon, e.g., tectonic setting. The fact that (37) combined with the four other constraints mentioned above is consistent with the observed quasi-universal relations for hydraulic geometry suggests, however, a strong element of self-similarity in catchment organization.

11. Conclusions

[69] A baseline data set consisting of stream reaches from Alberta, Canada, Idaho, USA, Britain and the Colorado River, Colorado, USA is used to determine dimensionless bankfull hydraulic relations for alluvial, single-thread gravel bed streams with definable channels and floodplains. These dimensionless relations show a considerable degree of universality. Application of the regression relations to three other data sets, one from Maryland, USA, one from Colorado, USA and one from Britain, confirms this tendency toward universality. The relations are, however, only quasi-universal in that some systematic deviation from universality can be detected.

[70] The regression relations are used to back calculate the coefficients and exponents of a set of physical relations governing bankfull hydraulic geometry. This back calculation results in (1) a Manning-Strickler relation for channel resistance, (2) a relation in which the critical Shields number for the onset of gravel motion varies weakly with dimensionless flow discharge and (3) a relation for "gravel yield" which relates the dimensionless gravel transport rate at bankfull flow to dimensionless bankfull discharge. Within the framework of the analysis the relations for bankfull hydraulic geometry and the underlying physical relations are completely equivalent to each other. In this way a solution to the basis for the relations for hydraulic geometry is obtained without invoking an extremal hypothesis.

[71] The underlying physical relations allow for generalization of the coefficients in the hydraulic relations in such a way that the effects of varying "bank strength," channel resistance and gravel supply on hydraulic geometry can be estimated. The Manning-Strickler resistance relation back calculated from the data provides a means for estimating bankfull discharge from measured values of bankfull depth, bankfull width, down-channel bed slope and surface median size. The resistance relation performs well against both the baseline data set and the data sets from Colorado, Maryland and Britain that were not used to determine the relation.

[72] The analysis allows an estimation of the effect of form drag in gravel bed streams at bankfull flow. This estimation suggests that form drag becomes progressively more important as the ratio of bankfull depth to surface median size decreases.

[73] Finally, the analysis suggests that the piece of information missing from previous analyses to close the formulation for bankfull hydraulic geometry is not some kind of extremal constraint applied to a cross section, but rather a relation that expresses how a catchment organizes itself to deliver gravel downstream, i.e., a "gravel yield" relation.

Appendix A: Dimensionless Variables and Spurious Correlation

[74] The parameters used in the analysis given above are not the dimensioned parameters that are measured in the field, but rather dimensionless groupings of these parameters. For example, rather than searching for a relation between bankfull width B_{bf} and bankfull discharge Q_{bf} a relation is sought between $\tilde{B}$ and $\hat{Q}$, i.e.,

$$\tilde{B} = \hat{f}_{\tilde{B}}(\hat{Q}) \tag{A1}$$

where

$$\tilde{B} = \frac{B_{bf} g^{1/5}}{Q_{bf}^{2/5}} \tag{A2a}$$

$$\hat{Q} = \frac{Q_{bf}}{\sqrt{g D_{s50}}\, D_{s50}^2} \tag{A2b}$$

In so far as the parameter Q_{bf} appears in both the dependent and the independent parameter, any correlation between the two is open to the criticism that it might be spurious [*Benson*, 1965].

[75] Such criticism can be misplaced in the case of dimensionless parameters. When formulating a problem in terms of dimensionless groupings, it is often inevitable that the same dimensioned parameter appears on both sides of the equation. Consider the case of the resistance relation for

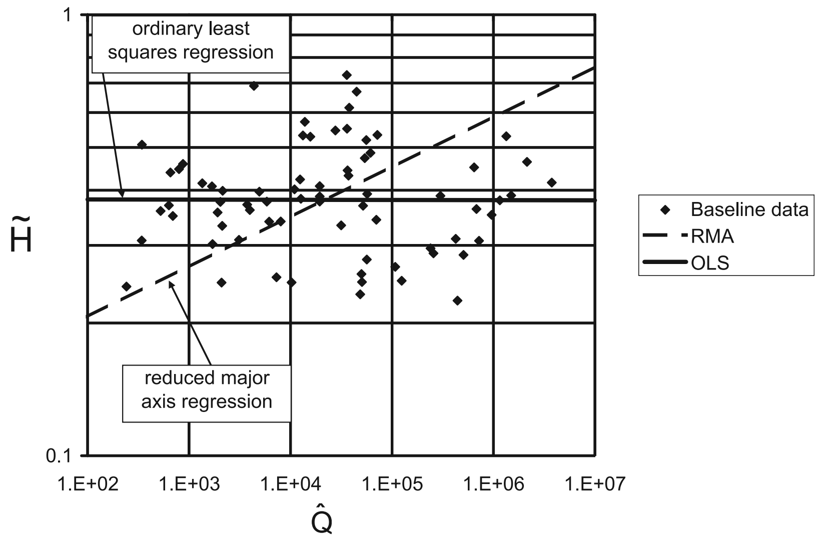

Figure B1. Plot of $\tilde{H}$ versus $\hat{Q}$ using the baseline data set. Also shown is the power relation obtained with ordinary least squares (OLS) regression, as well as the corresponding one obtained with reduced major axis (RMA) regression.

steady, laminar flow in a round pipe. The boundary shear stress τ_b at the pipe wall depends upon fluid density ρ, fluid kinematic viscosity ν, cross-sectionally averaged flow velocity U and pipe diameter D. Simple dimensional analysis applied to a list of five parameters (τ_b, ρ, ν, U, D) containing three dimensions (length L, time T and mass M) indicates that any relation between the five dimensioned parameters can be reduced to one involving exactly two dimensionless parameters [e.g., *Kundu and Cohen*, 2002].

[76] It is impossible to construct two dimensionless parameters from the stated list without at least one dimensioned parameter appearing in both. A clear case in point involves the D'arcy-Weisbach friction coefficient *f* and the Reynolds number **Re**:

$$f = \frac{\tau_b}{\frac{1}{8}\rho U^2} \tag{A3a}$$

$$\mathbf{Re} = \frac{UD}{\nu} \tag{A3b}$$

Note that the flow velocity U appears in both parameters. Now let us assume that *f* and **Re** were determined from measured values of τ_b, ρ, ν, U and D, all of which are subject to measurement error. A regression between *f* and **Re** in the form of a power relation would yield the form

$$f = a\mathbf{Re}^n \tag{A4}$$

where a and b would be determined from the regression analysis.

[77] According to *Benson* [1965], the power correlation between *f* and **Re** of (A4) is subject to spurious correlation and thus should be discarded. Yet an exact solution of the Navier-Stokes equations yields the form

$$f = \frac{64}{\mathbf{Re}} \tag{A5}$$

[e.g., *Potter and Wiggert*, 2002], i.e., a = 64 and n = −1.

[78] If we accept for a moment that *f* cannot be regressed against **Re** because of the possibility of spurious correlation, then what are the alternatives? One possibility is to regress τ_b against some combination of the other four parameters which has the same dimensions, i.e., ($\rho\nu$U/D) in the present case. Thus the power relation sought would be

$$\tau_b = a_1 \left(\frac{\rho\nu U}{D}\right)^{n_1} \tag{A6}$$

The correct values of a_1 and n_1 corresponding to (A5) are $a_1 = 8$ and $n_1 = 1$.

[79] Any error at all in the measured parameters all but guarantees that the exponent n_1 in (A6) differs from 1. A value of n_1 differing even slightly from 1 in (A6) in turn forces the value of a_1 to have dimensions that vary with the value of n_1. For example, if the value of n_1 determined from regression were found to be 1.06 rather than the exact value of unity, then the dimensions of a_1 that are required to preserve dimensional homogeneity in (A6) are $M^{-0.06}L^{0.06}T^{0.12}$. Each experimental data set is likely to result in somewhat different dimensions for a_1. Such error-dependent dimensions attached to a_1 imply that the method has not adequately captured the underlying physics.

[80] The form (A4) does not have this problem. The values of a and n determined by regression may not be precisely equal to the theoretical values of 64 and −1, respectively, but they nevertheless remain dimensionless, and thus capture the underlying correlation between two dimensionless parameters, *f* and **Re**, embodied in the Navier-Stokes equations.

Appendix B: Ordinary Least Squares Regression and Reduced Major Axis Regression

[81] Consider a set of measured parameters (x, y). One way to determine a relation correlating y (dependent variable) to x (independent variable) is by means of ordinary least squares (OLS) regression. This method is particularly

appropriate when y is subject to measurement errors but x is not. When both y and x are subject to measurement errors, reduced major axis (RMA) regression is often a better alternative [*Mark and Church*, 1977]. This is because the regression line obtained from RMA tends to fall between the line obtained by regressing y against x using OLS and the line obtained by regressing x against y using OLS.

[82] All of the parameters used in the present analysis are subject to measurement error. With this in mind, it might be thought that RMA is preferable to OLS. This turns out not to be the case. In particular, when y is sufficiently poorly correlated to x to indicate a relation of the form $y \sim x^0$, RMA tries to split the difference between this relation and the relation $x \sim y^{\infty}$, so yielding erroneous results.

[83] Specifically this case arises in the regression of $\tilde{H}$ against $\hat{Q}$. As reported above, the OLS regression using the baseline data set is

$$\tilde{H} = 0.382 \quad \hat{Q}^{-0.0004} \tag{B1}$$

The corresponding RMA regression is

$$\tilde{H} = 0.122 \quad \hat{Q}^{0.113} \tag{B2}$$

A perusal of Figure B1 should convince the reader that the result obtained from OLS is the more appropriate one. For the sake of consistency OLS has been used throughout this paper.

Notation

B_{bf} bankfull width.
$\hat{B}$ $= B_{bf}/D_{s50}$.
$\tilde{B}$ $= g^{1/5} B_{bf}/Q_{bf}^{2/5}$.
C_f resistance coefficient.
C_{fs} coefficient of resistance due to skin friction.
D_{s50} bed surface size such that 50% are finer.
D_{s90} bed surface size such that 90% are finer.
g gravitational acceleration.
H_{bf} bankfull depth.
$\hat{H}$ $= H_{bf}/D_{s50}$.
$\tilde{H}$ $= g^{1/5} B_{bf}/Q_{bf}^{2/5}$.
k_s bed roughness height.
n_B exponent in dimensional hydraulic relation (1a) or dimensionless hydraulic relation (8a).
n_H exponent in dimensional hydraulic relation (1b) or dimensionless hydraulic relation (8b).
n_r exponent in (20a).
n_S exponent in dimensional hydraulic relation (1c) or dimensionless hydraulic relation (8c).
n_y exponent in (26a).
n_τ exponent in (24).
U_{bf} mean flow velocity at bankfull flow.
$u_{*,bf}$ shear velocity at bankfull flow.
Q_{bf} bankfull discharge.
$\hat{Q}$ $= Q_{bf}/(\sqrt{gD_{s50}}\ D_{s50}^2)$.
$Q_{b,bf}$ volume bed load transport rate at bankfull flow.
$\hat{Q}_b$ $= Q_{b,bf}/(\sqrt{gD_{s50}}\ D_{s50}^2)$.
Q_2 flood discharge with a two-year recurrence interval.
q_{bf}^* dimensionless Einstein number characterizing gravel transport rate at bankfull flow, defined in (18b).
R $= (\rho_s - \rho)/\rho$; submerged specific gravity of sediment.
r ratio between bankfull Shields number and critical Shields number, defined in (22).
S channel bed slope.
$(X)_{pred}$ predicted value of any parameter X.
$(X)_{rep}$ reported value of any parameter X.
α_B dimensionless coefficient in (8a).
α_G dimensionless coefficient in (21).
α_H dimensionless coefficient in (8b).
α_r dimensionless coefficient in (20a).
α_S dimensionless coefficient in (8c).
α_y dimensionless coefficient in (26a).
α_τ dimensionless coefficient in (24).
φ_f fraction of resistance that is form drag, defined in (50).
$\tau_{b,bf}$ bed shear stress at bankfull flow.
τ_{bf}^* dimensionless Shields number at bankfull flow, defined in (18a).
τ_c^* dimensionless Shields number at the threshold of motion.
ρ_s material density of sediment.
ρ density of water.

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