Eolian dunes: Computer simulations and attractor interpretation

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ABSTRACT

A simple computer-simulation algorithm for the transport of sand by wind produces forms resembling barchan, crescentic ridge, linear, and star natural dune classes. Sand is moved as slabs composed of many grains that are picked up at random, transported in a specified direction, and deposited (1) with a probability that depends on the local presence or absence of sand or (2) in shadow zones in the lee of dunes. The simulated dune fields are interpreted as complex systems, with sand-dune classes being dynamical attractors of these systems. The evolution of dunes once formed becomes decoupled from the details of eolian sand transport.

INTRODUCTION

Many ideas and models have been advanced to explain the morphology of eolian dunes in terms of the processes governing their initiation and evolution (reviewed by Cooke et al., 1993). Because of the long time required for full development, the evolutionary sequence has been documented in only a few cases (Kocurek et al., 1992). A time-dependent model relating wind and sediment parameters to dune morphology would be helpful for addressing questions regarding the efficacy of using the morphology of existing dune fields as paleo–climatic indicators and for inverting the eolian rock record to determine the details of past environmental conditions. It also would complement an existing model that relates dune morphology and orientation to stratigraphy (Rubin, 1987).

Because of the complicated nature of air flow and of sand erosion, transport, and deposition over a dune, a reductionist path from the physics of eolian sand transport to the evolution of a dune field is not currently feasible. Most of the models that have been proposed for eolian sand transport and dune formation and evolution suffer from an empirical nature that inhibits their wide application, an inability to lead to testable hypotheses on the dune-field scale, or a lack of generality that limits consideration to highly specialized situations rarely encountered in nature. No fundamental, unifying theories have emerged, and no path to an all-encompassing framework has been identified.

A robust method for simulating the development and evolution of eolian dune fields is presented here. The goal is to introduce the technique, demonstrate that it produces forms that qualitatively resemble natural dunes, and apply a tentative interpretation to its results. The justification for this technique is grounded in two properties of systems subject to strongly nonlinear, dissipative processes (e.g., Nicolis and Prigogine, 1989). (1) These systems commonly evolve to a finite number of steady states, termed *attractors*, from a broad range of initial conditions. The existence of attractors often can simplify the dynamical description of the system to treatment of the approach to and transition between attracting states. (2) *Emergent behavior* is self-organizing behavior on macroscopic space or time scales that is consistent with the microscopic physics, but is both simpler and essentially decoupled from the small scale. Systems exhibiting these two properties fall into a class of nonlinear systems termed *complex systems*, for which complicated behavior (such as deterministic chaos) results from simple underlying physics. Eolian sand transport is both nonlinear (e.g., grain flux depends nonlinearly on wind velocity) and dissipative (e.g., energy loss in grain collisions). If dune-forming systems are complex systems, these two properties permit considerable simplification in constructing an algorithm for transporting sand and building dunes.

ALGORITHM

In the computer simulation, dunes are built from slabs of sand, the positions of which are constrained to lie on a square lattice (Fig. 1). The surface elevation is proportional to the number of sand slabs at a lattice site. The edges of the lattice are connected by periodic boundary conditions, whereby a slab of sand transported over one boundary of the lattice is brought in at the same position on the opposite boundary. A maximum difference in surface elevation between adjacent lattice sites, an angle of repose set to 30°, is enforced. If the deposition of a slab of sand at a particular site would violate the angle-of-repose criterion, the slab is moved down the steepest gradient until compliance is achieved. Similarly, if erosion of a sand slab oversteepens the slope, starting at that lattice site, neighboring sand slabs are moved downslope one lattice site, successively going up the steepest gradient until no angle exceeds the angle of repose.

For some of the simulations, the initial morphology is generated by placing sand slabs, one by one, on the lattice at random locations. Alternatively, dunes with a range of configurations and orientations are constructed as initial conditions.

Sand slabs are transported individually in a manner that is meant to approximate the conveyance of sand grains in saltation by the wind over sand beds and nonerodible, rough surfaces. A sand slab is chosen for transport (erosion) randomly from all sand slabs on the surface. The slab is moved a specified number of lattice sites, l, in the transport direction and is deposited at this site with a probability that depends upon the number of sand slabs there. The probability of deposition at a site with no sand slabs, p_{ns}, is less than the probability of deposition at a site with at least one sand slab, p_s. This differentiation stems from the presumed greater likelihood of rebound for saltating grains from a stony surface vs. a sand patch (Bagnold, 1973). If the slab is not deposited, then it repeatedly is moved l sites in the transport direction until deposition, following which another slab is chosen randomly for transport. This procedure is repeated to construct the time evolution of the surface. Slab movement not parallel to the transport direction originates only from enforcement of the angle of repose.

Time t in the simulations is the number of lattice sites that have been polled for slab erosion divided by the number of surface lattice sites in the simulation. Simulated time intervals can be related to actual time intervals by specifying the physical dimensions of the slab and then scaling time so as to achieve a given mean sand flux or a given mean dune-migration speed.

RESULTS

Application of the simple algorithm outlined above leads to the formation of simulated dunes and dune fields with morphology and behavior closely resembling those of several natural dune classes. The results of a simulation in which forms similar to barchan dunes develop are illustrated in Figure 2A. Under unidirectional transport

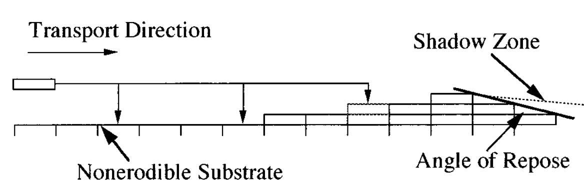

Figure 1. Side view illustrating dune-simulation transport algorithm.

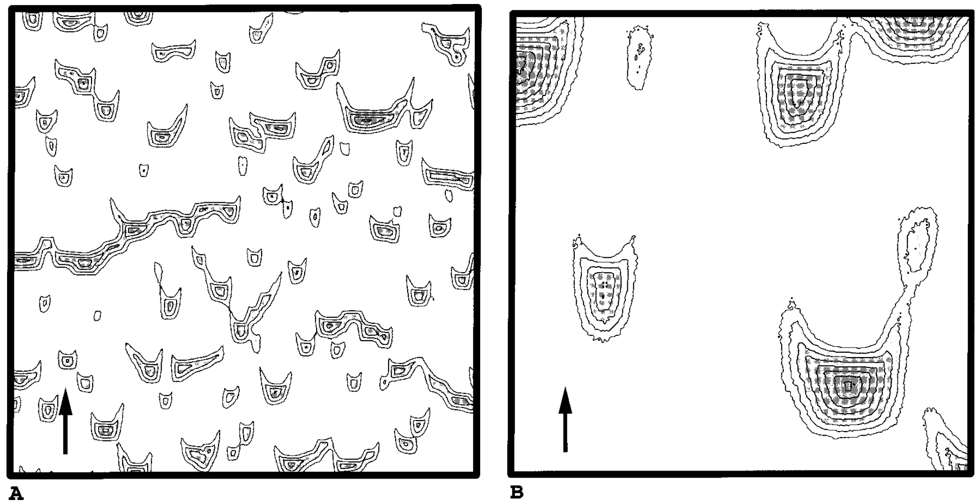

Figure 2. A: Simulated barchan dune field on 1000 × 1000 lattice at t = 500 from initial random morphology. Transport direction toward top of map, average of three slabs per lattice site, slab aspect ratio = 1/3, l = 5, p_s = 0.6, p_{ns} = 0.4, shadow zone 15° to horizontal (same aspect ratio, deposition probabilities, and shadow zone for all simulations). Contour interval = 10 slabs. B: Simulated barchan-dune calving in same simulation as A. Contour interval = 5 slabs.

and starting from random morphology, dunes with a horizontal spatial scale related to l, p_s and p_{ns} are spawned. The simulated dunes develop the characteristic shape of a barchan, with horns facing in the transport direction, because of the inverse relation between dune-migration speed and cross-sectional size. Dune-crest terminations (e.g., horns for barchans) move faster than the main body of the dune, pointing in the direction of transport so that the ratio of surface perimeter exposed to active transport to cross-sectional area is approximately constant and independent of lateral position along the dune.

The general development of barchans does not depend on the initial placement of the sand, although the precise number, shape, and positions of incipient dunes are sensitive to this placement. One exception is a simulation beginning with sand arranged into long transverse ridges. These ridges do not break up into barchan dunes.

The simulated dunes increase in size by merger between two dunes; a small dune catches up to a large dune, and the two coalesce. Dunes grow also by lateral linking; the horns of laterally adjacent dunes of different sizes coalesce as the smaller dune passes the larger. The simulated barchans continually lose sand from their horns. Dunes are found along the transport paths leading from the horns of other barchans so as to make up for this loss. Occasionally a small dune detaches from the horn of a barchan, a behavior that has been termed calving (Fig. 2B). These processes for dune development and spawning and the alignment and ordering of a dune field observed in the simulations have counterparts in natural dune fields (Bagnold, 1973; Kocurek et al., 1992). With sufficiently high sand supply, simulated dunes resembling crescentic ridges form transverse to the transport direction, evolving from initial barchans via lateral linking (Fig. 3A).

The development of transverse dunes (barchans or crescentic ridges) is not sensitive to the parameters employed in the simulations. Varying the transport length and the deposition probabilities primarily alters their initial size and migration speed. Simulated barchan dunes can form without lee shadowing, and transverse dunes can be generated with $p_s = p_{ns}$. Changes to the algorithm—including normally distributed transport perpendicular to the mean transport direction and erosion or deposition probabilities that decrease or increase with increasing elevation or surface tilt—do not alter the general features of simulated dune formation. These changes appear to affect primarily the cross-sectional shape of the dunes.

It has been hypothesized that the observed range of dune classes can be explained by recourse to a sequence of sand-transporting winds from differing directions or with differing strengths (Bagnold, 1973; Rubin and Hunter, 1987). Wind regimes that have been proposed for linear and star dunes were implemented in the simulations both to subject this hypothesis to a test (within this simple framework) and to test the ability of a simple dune-building algorithm to generate a wider range of dune shapes. According to one hypothesis, linear dunes develop in oblique reversing winds (Bagnold, 1973). This hypothesis for linear dunes was explored by applying a transport direction that varies from one side to the other, with a small mean component. Simulated linear dunes form that are aligned along the mean transport direction (Fig. 3B). Star dunes are believed to form under complicated wind regimes (Fryberger and Dean, 1979). Simulated dunes with some of the characteristics of star dunes, including arms radiating from a central core, develop when the transport direction is rotated around the points of the compass (Fig. 3C).

INTERPRETATION

The simulation results reported here are suggestive of a general framework for eolian dunes wherein the bulk of the general features and behavior are describable in terms of very simple, general properties of the transport. Specifically, the results are consistent with the overall hypothesis of eolian dune fields as complex systems and with two particular hypotheses: (1) eolian dune classes (barchan, crescentic, linear, etc.) are attractors of the eolian sand transport and morphology system and (2) the evolution of dunes can be described as emergent behavior.

An attractor is a subset of the phase space (space of variables describing the state of a system) available to a system to which it contracts, owing to dissipation, from a broader region of phase space, its *basin of attraction*. Therefore, if simulated eolian dune classes are attractors, it should be possible to observe the evolution of the system to a particular dune class from a range of initial conditions. To characterize the simulated dune-forming system, two phase-space variables were chosen: dune orientation angle relative to the mean transport direction and the number of dune-crest terminations. Orientation is one variable that clearly distinguishes different dune classes. Number of terminations also distinguishes dune classes (e.g., crescentic vs. barchan) and can be a measure of the maturity of the dune pattern (e.g., the number of terminations decreases dramatically as simulated linear dunes form).

For a sequence of transport directions that resulted in simulated linear dunes (Fig. 3B), simulations were performed starting from well-formed dunes with orientations ranging from transverse to longitudinal and with 0 to 70 total dune-crest terminations. The simulated dune systems evolve to or near to the point in phase space

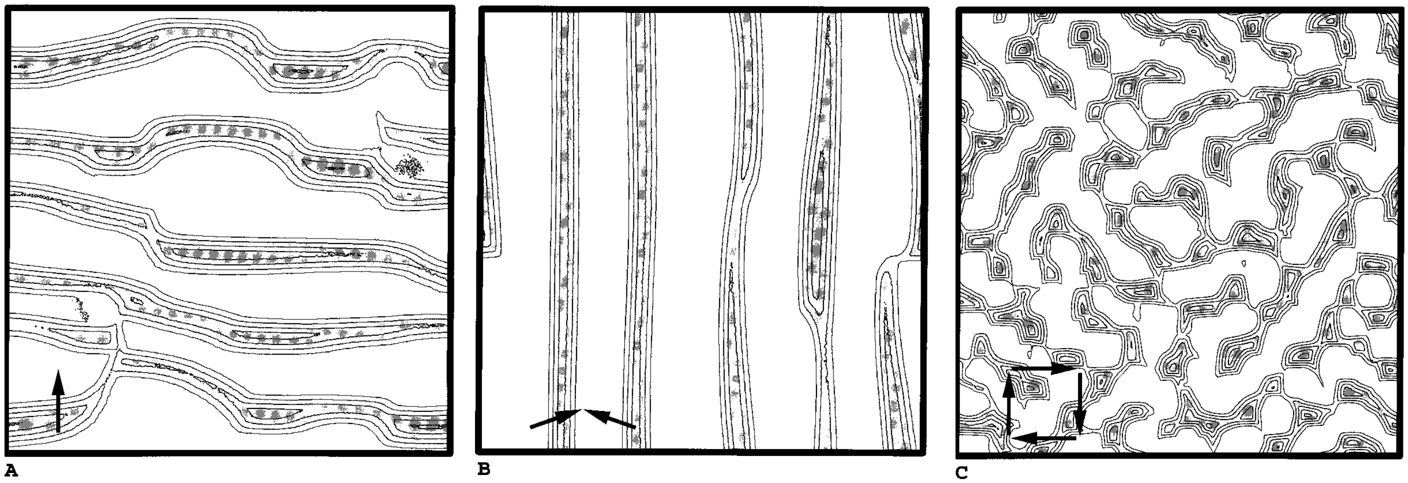

Figure 3. Simulated dune fields under differing sequences of transport direction evolving from initial random morphology. A: Simulated crescentic dunes at t = 500. Transport direction toward top of map, average of 15 slabs per lattice site, l = 5. Contour interval = 10 slabs. B: Simulated linear dunes at t = 2000, mean transport direction toward top of map, average of eight slabs per lattice site, l_1 = (2, 6) and l_2 = (2, −6) (horizontal, vertical), duration Δt = 40. Contour interval = 10 slabs. C: Simulated star dunes at t = 1600; average of 15 slabs per lattice site; l_1 = (5, 0), l_2 = (0, 5), l_3 = (−5, 0), and l_4 = (0, −5); duration Δt = 80. Contour interval = 10 slabs.

representing well-formed linear dunes, with orientation 0° and 0 terminations (Fig. 4). This state has the same characteristics as the state arising from a random initial condition under the same sequence of transport directions. This result is consistent with the existence of a single attractor for the system with a basin of attraction that is the entire phase space.

The nature of the hypothesized attractor varies abruptly with changes to the parameters of the model. For example, the asymptotic orientation changes from longitudinal to transverse as the obliquity of the reversing transport directions decreases below about 45° (Fig. 5). A similar but less sharp transition between barchans and crescentic ridges is observed as the sand supply is increased. Some tentative evidence for the simultaneous existence of two attractors for a single sequence of transport directions has been observed. In this case, the simulated dunes approach a longitudinal or transverse orientation depending on initial dune orientation.

Tests for the occurrence of emergent behavior in the simulations involve finding macroscopic dune behavior that is insensitive to the details of the microscopic transport rules employed; such tests are more qualitative than tests for attractors. The evolution of a simulated dune field is dominated by interactions between dunes that take several simple forms. First, because of the inverse relation between migration speed and size, small dunes catch up with large dunes and interact by sand transfer or merger. Second, simulated dunes lose sand from their own terminations and gain sand from properly aligned terminations of other dunes. Third, the focal points for interactions among long-crested dunes are the dune-crest terminations, which (for transverse dunes) migrate faster than the dunes because of their smaller cross-sectional size. The dynamics of terminations in simulated dune fields resemble the dynamics documented for natural (Anderson and McDonald, 1990) and simulated (Landry and Werner, 1994) wind ripples. A vivid illustration of emergent behavior is calving from the horns of barchan dunes (Fig. 2B). All of these behaviors depend only on the most general features of the transport (e.g., dune migration speed varying inversely with size) and have been reproduced by using many diverse transport rules (e.g., erosion or deposition that depends in various ways on elevation, slope, or lee shadowing).

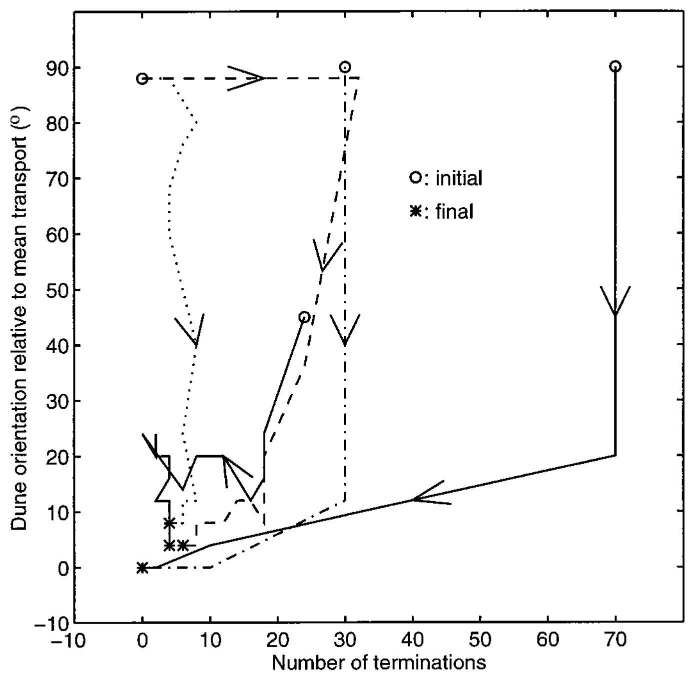

Figure 4. Hypothesized simulated linear-dune attractor. Sample phase-space trajectories starting from well-formed dunes with range of orientations and numbers of terminations and evolving to the region of 0° orientation and 0 terminations. Orientation was chosen to be the peak of the distribution of projections of the vectors tangent to a contour (number of slabs = 10) onto a line rotated in 4° increments. Number of terminations was counted by eye. Same transport parameters as Figure 3B.

DISCUSSION

The simple transport algorithm described above results in forms that resemble natural dunes under sequences of transport directions that are consistent with proposed hypotheses and field observations. The justification for this gross simplification of the dynamics of eolian sand transport originates in the nonlinear, dissipative nature of the dynamics, which gives rise to the expectation that dunes exhibit the property of emergent behavior of a complex system, a simplification of the dynamics for large space scales and long time scales.

Dunes have been classified both by their morphology (principally the geometry and topology of dune crests) and by their orientation based on observational evidence (Cooke et al., 1993). A

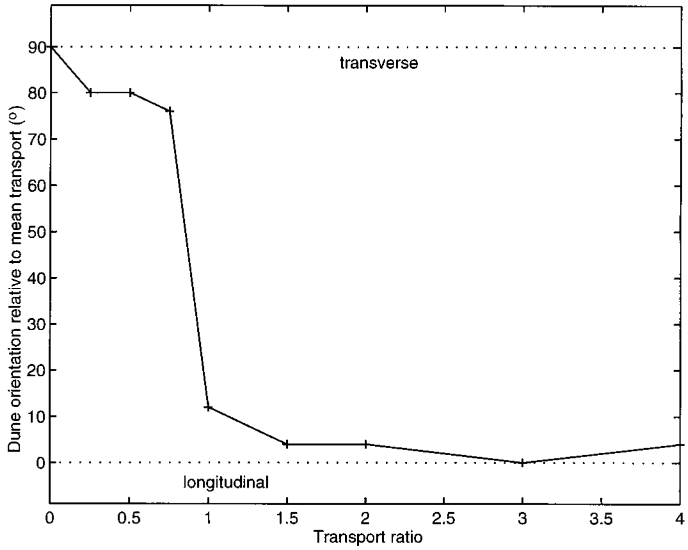

Figure 5. For oblique reversing transport, simulated dune orientation as function of ratio of transport normal to and parallel to mean transport direction. As transport varies from unidirectional to bidirectional, dune orientation changes from transverse to longitudinal at a transport ratio of ~1. Initial morphology in all simulations was random, average number of slabs = 5, and orientation measured at *t* = 1200.

plausible hypothesis is that these classifications correspond to the attractors of the eolian sand transport system. In this view, the variables that capture the interesting dynamical behavior of the system (including dune orientation and number of terminations) are those variables for which evolution to an attractor occurs on the time scale associated with this interesting behavior. The vast majority of variables, which evolve faster, do not influence the dominant behavior and are said to be slaved to these interesting variables.

The results and provisional interpretations presented here should be viewed as tentative because of (at least) four criticisms that may require further investigation. First, because the treatment of aerodynamical effects is highly simplified, the initiation of a dune until the development of a slipface probably is not modeled well (except perhaps for calving, where sand supply, rather than aerodynamics, may be dominant). Second, the use of periodic boundary conditions can artificially influence the evolution of simulated dunes. Increasing the size of the periodic domain does not alter qualitative dune characteristics (general orientation and morphology) for the simulations described above. Third, the simple rules for erosion and deposition employed result in somewhat steep stoss slopes and a weak but unphysical dependence of cross-sectional shape on dune size. A possible remedy is modifying the transport rules to account for the relationships among local morphology, wind profile, and transport flux in greater detail. Fourth, the variables *orientation* and *number of terminations* do not describe simulated dunes adequately as a dynamical system, as is seen from the nonuniqueness of the phase-space trajectories emanating from a single point (Fig. 4). At least one more variable, possibly characteristic dune size, is required to specify the macrostate of dune systems.

The simulation algorithm described here is potentially useful in that it can make testable predictions for complicated three-dimensional morphologies and for complicated wind regimes. This characteristic permits falsification of the assumptions of the model through direct comparison to field data. In addition, the algorithm may be useful for exploring both the consequences and dynamical origin of the numerous hypotheses that have been advanced regarding eolian dunes. For example, it should be possible to investigate a fundamental dynamical explanation for Rubin and Hunter's (1987) hypothesis that dunes are oriented so as to maximize sand transport normal to the dune crest and to test its consistency with the simulation algorithm. Finally, mapping out the range of transport regime parameters and initial conditions for particular dune-class attractors could allow careful quantification of the uncertainty in inverting dune shapes and orientations for wind regimes and initial conditions.

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